Physics III: Final (Group)

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(Tuesday July 18, 10:25 AM)

First and Last Name: ______________________________

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Exam Instructions:
This is an open-notebook exam, so feel free to use the notes you have transcribed throughout the summer and problem sets you have completed, but cellphones, laptops, and any notes written by someone else are prohibited. You will have **40 minutes** to complete this exam.

Since this is a timed exam, your solutions need not be as ”organized” as are your solutions to assignments. Short calculations and succinct explanations are acceptable, and you can state (without derivation) the standard solutions to the equations of motion we derived in class. However, you should also recognize that you cannot receive partial credit for derivations/explanations you do not provide.

Problem 1: _______________

Problem 2: _______________

Total:

1. **Rolling in bowl (30 points)**
A spherical ball of radius r and mass M, moving under the influence of gravity, rolls back and forth without slipping across the center of a bowl which is itself spherical with a larger radius R (Fig. 1). The position of the ball can be described by the angle θ between the vertical and a line drawn from the center of curvature of the bowl to the center of mass of the ball.

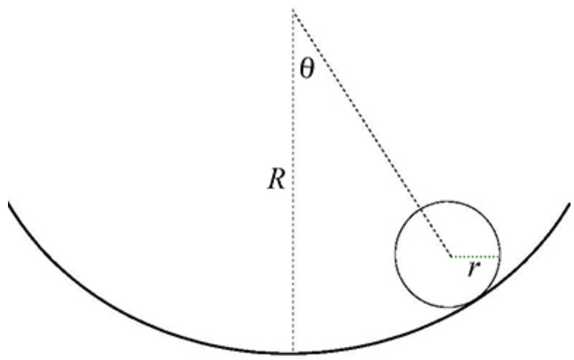

Figure 1: Ball in Bowl

The total energy of the system is

$$E_{\text{tot}} = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} M v^2 + M g (R - r)(1 - \cos\theta), \tag{1}$$

where $I = \frac{2}{5} M r^2$ is the moment of inertia of the ball, and $v = r\dot{\theta}$ is the velocity of the sphere. All quantities except θ are time-independent constants.

(a) (10 points) Assume that the ball begins from rest at an angle θ_0 away from the vertical. Using conservation of energy, derive an expression for the period of the ball (i.e., the time it takes the ball to move from θ_0 to $-\theta_0$ and back to θ_0.)

(b) (10 points) By conservation of energy, E_{tot} must be independent of time. Using this fact, Eq.(1), and what you know about derivatives derive an equation of motion for θ.

(c) (10 points) Take the small-angle approximation for the equation derived in (b). What should the result in (a) reduce to in this approximation?

[Scratch work]

2. **Coupled Oscillator (30 points)**
Three masses are coupled through two springs of spring constant as shown in the figure below.

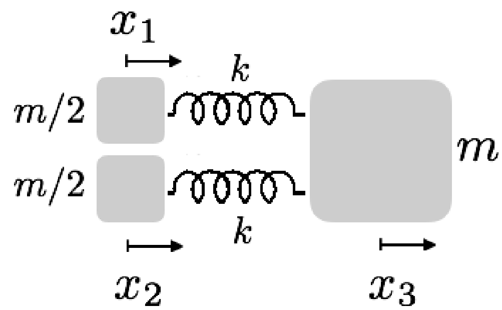

Figure 2: Three Oscillators

The two smaller masses each have mass $m/2$, the larger mass has mass m, and all the springs have spring constant k.

(a) (10 points) The system has three normal modes. One of these modes consists of the all of the masses moving to the right at constant speed, i.e.,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{2}$$

Using symmetry and the fact that the center of mass of the two other types of motion remains constant, write the other two normal modes as vectors of the form

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}, \tag{3}$$

where you should determine A, B, and C.

(b) (10 points) Write the equation of motion of the above system as a matrix equation.

(c) (10 points) What are the normal mode **frequencies** of the system? *Hint: One of these frequencies is very simple*

[Scratch work]