Physics III: Midterm (Individual)

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(Friday June 30)

First and Last Name: ________________________________

Exam Instructions:
This is an open-notebook exam, so feel free to use the notes you have transcribed throughout the summer and problem sets you have completed, but cellphones, laptops, and any notes written by someone else are prohibited. You will have **1 hour** to complete this exam.

Since this is a timed exam, your solutions need not be as ”organized” as are your solutions to assignments. Short calculations and succinct explanations are acceptable, and you can state (without derivation) the standard solutions to the equations of motion we derived in class. However, you should also recognize that you cannot receive partial credit for derivations/explanations you do not provide.

Also, **there are 75 possible points on this exam, but the exam will only be graded out of 60 points.** Thus, be mindful of the time you're spending on each problem.

Problem 1: ________________

Problem 2: ________________

Problem 3: ________________

Problem 4: ________________

Total:

1. **Short Calculation Questions (15 points)**
   Answer the following questions, including a calculation justifying your answer. (You don't need to provide an elaborate explanation)

   (a) (2 points) A vertical spring on which is hung a block of mass m_1 oscillates with angular frequency ω. With an additional block of mass $m_2 \neq m_1$ added to the spring, the frequency is $\omega/2$. What is the ratio m_1/m_2?

   (b) (2 points) Two horizontal springs have identical spring constants, but one has a ball of mass m attached to it and the other has a ball of mass $2m$ attached to it. If the energies of the two systems are the same, what is the ratio of the oscillation amplitudes?

   (c) (2 points) You stop your car to pick up a member of your car pool. After she gets in, does the angular frequency ω of the oscillation due to the car's suspension, increase, decrease or stay the same?

   (d) (2 points) Given an object suspended by a spring, which of these variables of the motion can you control by varying the initial conditions: period, amplitude, energy of the system, frequency, phase, maximum velocity, maximum acceleration

   (e) (2 points) We have the potential energy function

$$U(x) = B\left(\frac{x}{A} + \frac{A}{x}\right), \tag{1}$$

   where x defines distance. What are the units of B? What are the units of A?

   (f) (2 points) For the above $U(x)$: At what value of x would a particle of mass m in the potential be at a stable equilibrium?

   (g) (3 points) For the above $U(x)$: The particle begins at rest a distance d away from the stable equilibrium position, where d is sufficiently small that simple harmonic motion is possible. The particle's initial energy is E_0. What would the energy be if A is changed to $2A$?

2. **An Angled Rail (20 points)**
Say we have a particle m constrained to move along a rail that makes a fixed angle θ with the vertical. The particle is attached to a spring of spring constant k which is itself attached to a wall by a sliding attachment such that the attachment is always at the same height as the mass and the spring is always horizontal. Say the particle is at a stable equilibrium when it is at the position shown in Fig. 1.

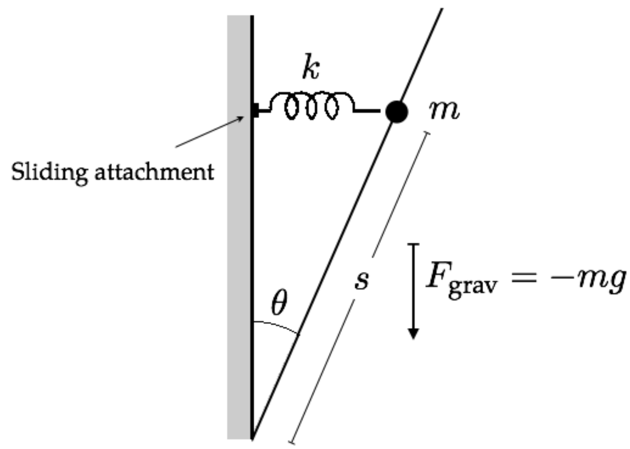

Figure 1: Angled Rails

We track the position of the particle with $s(t)$, the distance between the particle and the wall-rail attachment point.

(a) (5 points) If we set $\theta = \pi/2$, the mass only moves horizontally. In such a scenario, the mass has an equilibrium position $s = X_{\text{eq}}$. What is the equilibrium position for the general θ shown in the figure?

(b) (5 points) What is the equation of motion of the system in terms of s? What should the equation of motion be if we take $\theta \to 0$? Check that your two answers are consistent.

(c) (10 points) Say the particle is displaced from the equilibrium shown in Fig. 1 such that its total energy is E_0 and it first reaches the amplitude of its motion at time $t = 3/4\omega_0$ (where ω_0 is the angular frequency of oscillation). Find $s(t)$ as a function of time in terms of these initial conditions and the prior defined parameters.

3. **Maximum speed (20 points)**
A mass on the end of a spring (with natural frequency ω_0) is released from rest at position x_0. The experiment is repeated, but now with the system immersed in a fluid that causes the motion to be damped (with damping coefficient $\gamma = b/2m$).

In terms of the above defined parameters:

(a) (5 points) For the case of critically damped motion, what is $x(t)$?

(b) (5 points) Again for the case of critically damped motion, at what time is the speed maximum ?

(c) (10 points) For the case of *overdamped* motion, what is $x(t)$?

4. **Forced Pendulum (20 points)**
Say we have an undamped pendulum consisting of a mass m attached to a string ℓ. The pendulum begins at rest from its equilibrium position, and at this initial time we apply a sinusoidal force $F(t) = F_0 \cos(\omega t)$ to the mass. This force is applied along the arc of the mass's trajectory that is, the force is tangential to the arc and has no radial components (as shown in Fig. 2).

Take the small-angle approximation throughout.

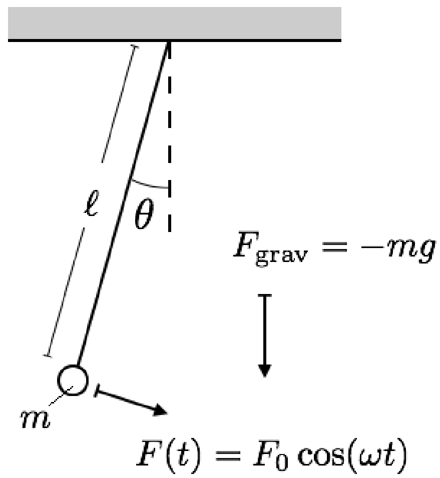

Figure 2: Driven pendulum

Assume throughout that the small angle approximation is valid.

(a) (5 points) What is the equation of motion of the system?

(b) (5 points) At what angular frequency would we need to apply the driving force in order for the system to be in resonance?

(c) (10 points) The mass begins at rest from the position $\theta = 0$. We apply the force with ω equal to the resonance frequency, with the system obeying the initial conditions stated in the prompt. Moreover, say that we can only apply the small-angle approximation when the amplitude of the motion θ_{amp} satisfies $\theta_{\text{amp}} < \frac{3}{4}$ radians. How much time would it take for the small angle approximation to not be a valid description of the system?