Assignment 1: Introduction and Kinematics Review

Due Wednesday June 14, at 9AM under Rene García's door

Preface: Before you begin writing up your solution to this assignment, read **in full** the supplementary note 01 on "Presenting work". In particular pay attention to the "Write Legibly" and "Explanations are part of the solution" bullet points. In short, you should draft your work so that someone who is not accostomed to your hand writing can read it, and you should include both **qualitative** explanations and **mathematical** derivations in your solution.

1. **Introduce Yourself**

   Tell me more about yourself. (You can detach this first page and staple it to the front of your assignment)

   (a) What city/state/country are you from?

   (b) What classes (both science and non-science) are you taking next semester/year?

   (c) Background: Next to the following topics put "1" if you have in the past solved 1-5 problems on the topic; "2" if you've solved 5-10 problems; and "3" if you've solved 10+ problems on the topic. Put "0" if you haven't solved any problems on the topic before.

   - Complex numbers
   - Trigonometry
   - Differentiation
   - Integration
   - Taylor series
   - Eigenvectors/Eigenvalues
   - Fourier Series
   - Newton's laws
   - Maxwell's equations
   - Phase Space

   (d) What skills/knowledge do you hope to gain from this class?

   (e) Anything else you want the instructor or the TA to know?

2. **Power Series and Perturbation Theory**
In physics, perturbation theory is a method for approximately solving a nonlinear equation (which typically is insoluble) in terms of its approximate linear solution. We will be encountering perturbation theory later in the context of oscillations, but we review the basic formalism here.

Say we want to solve the equation

$$\alpha x^2 - \beta = \varepsilon x^3 \tag{1}$$

for x, where α, β, and ε are numbers and $\varepsilon \ll \beta$ and $\varepsilon \ll \alpha$ ("$\ll$" is mathematics notation for "much less than" e.g., $10^{-3} \ll 1$).

(a) The first step of the perturbation theory procedure is to write the assumed solution x as a **power series** in the **perturbation parameter** ε:

$$x = x_{(0)} + \varepsilon\, x_{(1)} + \varepsilon^2\, x_{(2)} + \cdots , \tag{2}$$

where $x_{(k)}$ for $k \geq 0$ is called the ***k*th order correction** to the solution of x. Currently, $x_{(k)}$ for $k \geq 0$ is unknown. Insert Eq.(2) into Eq.(1) and expand the result up to ε^2 on both sides of the equal sign. What is the resulting equation?

(b) On both sides of the equal sign, equate the the coefficients of the ε-independent term, the coefficients of the ε^1 term, and coefficients of the ε^2 term. You should find a system of three equations. Use the resulting equations to determine $x_{(0)}$, $x_{(1)}$, and $x_{(2)}$ (in that order).

(c) Insert the values you found in the previous part into Eq.(2) to find the solution to Eq.(1) up to terms of order ε^2 or lower. You should find two solutions for x.
Considering Eq.(1), check that you get the expected two solutions when $\varepsilon \to 0$.

(d) Using your result in (d), calculate an approximation for the two real solutions of

$$x^2 - 1 = 0.1x^3. \tag{3}$$

Go to wolframalpha.com ➦ and type "Solve $x^2 - 1 = 0.1x^3$" into the search box. To three decimal places, what two real solutions do you get? How do they compare with the results of (d)?

3. **Marble and elevator**
Starting from $t = 0$, an elevator ascends from the ground floor with uniform speed. At time $t = T_1$ a boy drops a marble through the floor. The marble falls with uniform acceleration g, and hits the ground at $t = T_2$. Find the height of the elevator at time $t = T_1$. (*Hint: The velocity of the marble after it is released from the elevator is **not** zero.*) (Ans. check: When $T_2 = 2T_1 = 8$ sec, we find $h = 39.2$ m on earth.)

4. **Thought experiments in Kinematics**
A person throws a ball (at an angle of her choosing to achieve the maximum distance) with speed v from the edge of a cliff of height h. Assuming that one of the following quantities is the maximum horizontal distance d_{max} the ball can travel, which one is it and why?

$$\frac{gh^2}{v^2}, \quad \frac{v^2}{g}, \quad \sqrt{\frac{v^2 h}{g}}, \quad \frac{v^2}{g}\sqrt{1 + \frac{2gh}{v^2}}, \quad \frac{v^2}{g}\left(1 + \frac{2gh}{v^2}\right), \quad \frac{v^2/g}{1 - 2gh/v^2} \tag{4}$$

Explain why the other quantities are not valid. (**Do not** solve the problem completely, just check limiting cases. See supplementary note 03 "Dimensional Analysis and Limiting Cases" for a discussion of how to consider limiting cases.)

5. **Projectile Motion on a Hill**

Consider the physical situation depicted in the figure below. A rock is thrown at a speed v_0 and at an angle θ (from the horizontal) from the peak of a hill which slopes downward at an angle ϕ.

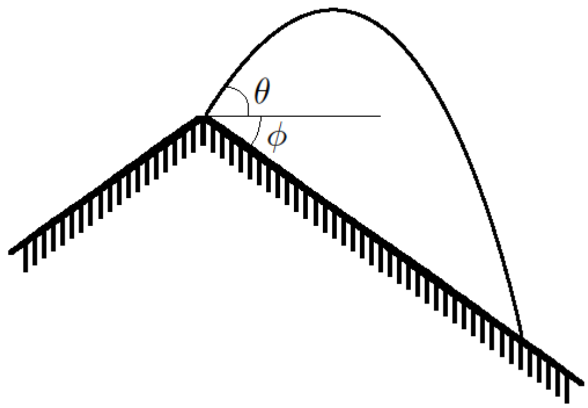

Figure 1: Projectile Motion

Write out **two** precisely stated physics questions, we could ask about this system. Answer both of your questions using your understanding of two-dimensional projectile motion.