Assignment 6: Electromagnetism and Final Exam Review Assignment

Problem 6 is due Sunday July 16, at 10PM under Rene García's door

Preface: The first part of this assignment consists of practice problems to review course material. The final problem provides an example of determining a magnetic field from an electric field. Use the first five problems as review (they are not to be handed in). **Only turn in Problem 6 on Sunday night.**

1. **Oscillations Near Equilibrium**
   A particle of mass m (confined to have position $x > 0$) is near the stable equilibrium of the potential

$$U(x) = \frac{\Delta_2}{x^2} - \frac{\Delta_4}{x^4}. \tag{1}$$

   What are the units of Δ_2 and Δ_4? If the mass begins at rest a distance ℓ_0 away from the stable equilibrium, what is the speed of the particle when it passes the equilibrium position?

2. **Underdamped Oscillator**
   An underdamped oscillator with phase $\phi = 0$ and initial amplitude A_0, starts off at the position $x(t = 0) = A_0$. The natural (i.e., undamped) frequency of the oscillator is ω_0 and the damping time constant is $\gamma = b/2m$ (with b the damping coefficient). At what time is the speed of the oscillator maximum? (Simplify result as much as possible)

3. **Forced Oscillator**
   A mass m is attached to a spring of spring constant k. The mass is at an equilibrium of the spring when it is at position $x = 0$. The mass begins from $x = 0$ with velocity v_0. Two forces $F_(1)(t)$ and $F_2(t)$ are applied to the mass as shown in Fig. 1. What is the position as a function of time $x(t)$?

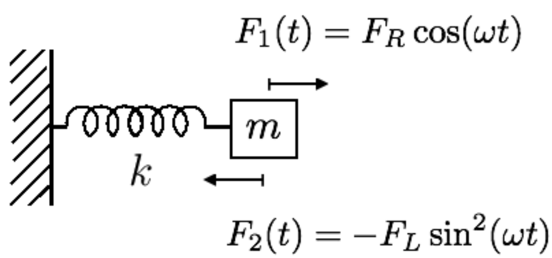

Figure 1

   What should we get as $\omega \to 0$? What should we get as $F_L \to 0$?

4. **Coupled Oscillator**
Two identical springs and two identical masses are attached to a wall as shown in Fig. 2. Find the normal mode (angular) frequencies and the corresponding normal modes of the system.

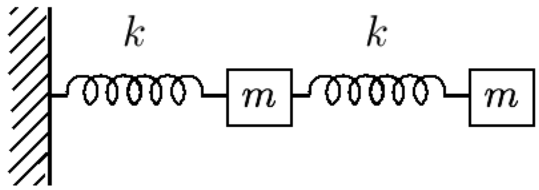

Figure 2

5. **Fourier Series and Waves**
A vibrating string, of mass density μ and tension T, has fixed ends. The string is confined to be within a domain of length L and begins at $y(x,0) = 0$ for all possible x in the domain. However, the string also begins with a transverse velocity given by

$$\dot{y}(x,0) = v_0 \sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right) + v_0 \sin\left(\frac{3\pi x}{L}\right). \tag{2}$$

What is $y(x,t)$ at time $t = t_1$ where

$$t_1 = \frac{L}{3}\sqrt{\frac{\mu}{T}}, \tag{3}$$

written as a function of x? (Simplify result as much as possible)

6. **Electromagnetism and Vector Calculus**
The electric field in a region of space is

$$\mathbf{E}(z,t) = E_0\Big(\cos(kz - \omega t)\,\hat{\mathbf{x}} + \sin(kz - \omega t)\,\hat{\mathbf{y}}\Big), \tag{4}$$

where E_0 has units of electric field, k is the wavenumber, and ω is the angular frequency.

(a) Using Faraday's Law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{5}$$

determine the magnetic field $\mathbf{B}(z,t)$ in this region of space. (Ignore any constants of integration)

(b) From your above results, compute $\mathbf{E} \cdot \mathbf{B}$.
(This shows that the electric and magnetic fields are perpendicular.)