

## Presenting Your Work

Just as important as learning how to solve physics problems is learning how to present and explain the problems you solve. In these notes we discuss the best practices for presenting your work in your solution sets. These practices are general and coincide with good writing habits for the final paper and any context where you are explaining mathematical arguments through writing. We also discuss why (and how) you should keep a notebook.

### 1 Mathematics in Writing

A relatively recent idea in education [1] argues that if you want to perform in a certain way, you should eventually evolve the qualities of your practice to being indistinguishable from the qualities of your desired performance. That is, you should train the way you want to someday execute.

In this course you will primarily develop your understanding of physics by solving problems, but in the solutions you submit you will also be developing the skills for written technical communication, and, in particular, the skills for writing and explaining mathematical arguments. These skills will be relevant for your final project where you and your group members will be explain through writing the solution to a multi-step physics problem. Therefore, it would be useful to lay down solid principles of writing practice now so that when the time comes for you to perform the appropriate communication skills, they are already familiar habits.

In these notes we discuss the best practices for writing up your solution sets, and although we focus on solution sets, the practices we discuss apply to any context where you are writing out a mathematical argument.

### 2 Writing Solutions

When writing your solutions to assignments there are a few habits of communication you should practice.

1. **Write Legibly with Organization:** It is important to recognize that someone who is not familiar with your handwriting will be reading your work, so you should take extra effort to ensure your characters and symbols are legible.

With regard to the organization of your solution, your work should follow a top-down logical progression in which one could draw a mostly vertical line from the start of a solution to the end. Having step 1 in the middle of the page, followed by step 2 in the upper-right-hand corner, and step 3 at the bottom of the page will, in general, be confusing.

Alternatively, instead of writing out your solutions on paper, you could use a  $\text{\TeX}$  typesetting software. (For example, this document was written using  $\text{\TeX}$ Shop.)

2. **Equations are Parts of Sentences:** Equations are parts of sentences and should be written with proper punctuation. Following this rule mostly comes down to including commas and periods where they are needed. For example, consider the following phrase and equation:

“Newton’s second law is

$$F = ma, \tag{1}$$

where  $F$  stands for force,  $m$  represents mass, and  $a$  represents acceleration.”

Note that there is a comma after the equation, and that if you were to read the entire sentence (including the equation) out loud, it would be grammatically correct and have correct punctuation. As a more elaborate example consider the passage taken from [2]:

The total revenue,  $R$ , made from selling widgets is given by the equation

$$R = pq, \tag{2}$$

where  $p$  is the price at which each widget is sold and  $q$  is the number of widgets sold. Based on past experience, we know that when widgets are priced at \$15 each, 2000 widgets will be sold. We also know that for every dollar increase in price, 150 fewer widgets are sold. Hence, if the price is increased by  $x$  dollars, then the revenue is

$$\begin{aligned} R &= (15 + x)(2000 - 150x) \\ &= -150x^2 - 250x + 30000. \end{aligned} \tag{3}$$

First note that complicated or important equations are given their own line. Note as well that each equation is followed by a punctuation mark. Equations which occur in the middle of sentences must always be followed by a comma, and equations which come at the end of sentences are always followed by a period.

3. **Explanations are Part of the Solution:** Physics is largely grounded in ideas and their corresponding mathematical expressions, and one component of translating the former into the latter is being able to track a mathematical argument from an initial question to a final answer.

Toward learning this skill, **single line answers are not sufficient in this course**. Your solutions should demonstrate that you know how to solve a problem in addition to knowing what the answer is. Indeed, “the real answer” to an assigned question isn’t only the single equation which specifically responds to the question but includes all the steps and arguments which led to that final equation.

It is also good practice to include not merely the steps of your derivation but words explaining your steps. You should imagine you’re explaining your work to someone who can follow the logic of an argument, but is not familiar with the solution. An example (transcribed from [3]) is useful for understanding the preferred types of solutions.

Say we have the following problem.

**Problem:** Find a point in the plane on the positive  $x$ -axis that has distance 5 from the point  $(2, 3)$ .

Consider a first possible “solution”.

**Solution #1 :**

$$x = 6$$

This solution is incomplete. Why?

- Answer is stated without derivation or explanation
- No sense of whether the student understands the answer

Next, let's consider a second solution.

**Solution #2 :**

$$\begin{aligned}(0 - 3)^2 + (x - 2)^2 &= 25 \\ 3^2 &= 9 + (x^2 - 4x + 4) = 25 \\ & x^2 - 4x - 12 \\ (x - 6)(x + 2) &\longrightarrow x = -2, 6 \quad x > 0 \quad x = 6\end{aligned}$$

This solution is more complete, but is also difficult to understand. Why?

- No reference to question being answered or the objective of the solution
- An unclear sense of what steps are being taken and virtually no sense of why they're being taken
- Improper use of equality on the second line
- Third line has an expression with no stated context.
- The final line uses an arrow to sloppily convert an expression to an equality, and the final results give mutually exclusive definitions for  $x$ .

Finally, let's consider a third possible solution.

**Solution #3 :**

Our goal is to find the value of  $x$  along the  $x$ -axis which is a distance of 5 away from the point  $(2, 3)$ . To find this, we note if  $(x, 0)$  is a solution, then  $x$  must satisfy the equation  $(x - 2)^2 + (0 - 3)^2 = 25$ , which follows from the distance formula between the points  $(x, 0)$  and  $(2, 3)$ . It follows that  $x^2 - 4x + 13 = 25$ . We then have

$$x^2 - 4x - 12 = 0. \tag{4}$$

Factoring, we obtain

$$(x - 6)(x + 2) = 0, \tag{5}$$

which is satisfied by either  $x = -2$  or  $x = 6$ . Since we assumed  $x > 0$  and  $y = 0$ , we see  $(6, 0)$  is the desired point.

This solution is complete and easy to understand. Why?

- The problem and plan for a solution are stated first.
- Equations are included in sentences with punctuation.
- Important equations are displayed by themselves and each equation is explained.
- The final answer occurs clearly at the end of the solution, and the assumptions/conditions which it depends on are plainly stated.

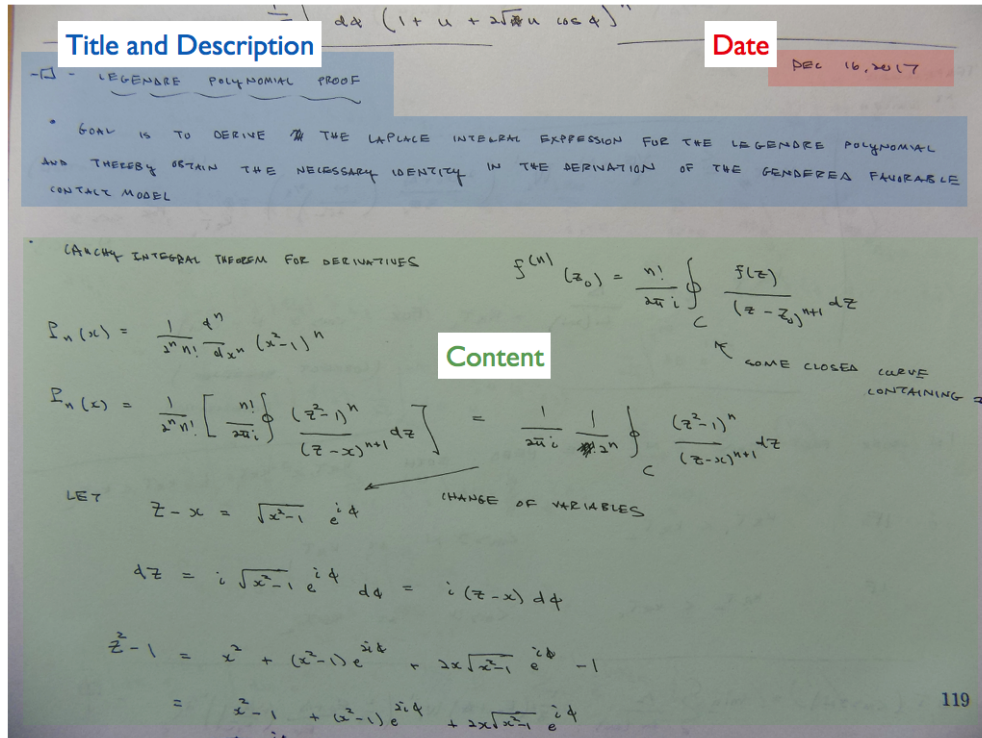


Figure 1: Example page from my (Mobolaji's) notebook. A date and a title are at the top of the section followed by a short description of the section's goals. Content comes after these preliminaries so that weeks later, instead of being confronted with a morass of symbols, I have a clearer sense of what I was trying to calculate and why.

### 3 Keeping a Notebook

Besides paying attention to the clarity and presentation of an argument, if there is one piece of advice I can give about writing solutions it is **to first try solving the problems in a personal notebook, before you write the solutions up for submission.** The other approach—of trying to solve the problem on the same page you intend to submit as a solution—will inevitably result in a disorganized and difficult to understand solution. This point deserves repetition:

1. Solve the problem in a notebook.
2. Think about how you want to organize the solution.
3. Write the solution on the paper intended for submission.

With regard to how you keep your notebook, it is completely your choice. I use a relatively simple format displayed below.

In general, I first write a title for the calculation or topic I'm working on, the date, a descriptive summary of the goal, and then I write the content of the calculation. It is of course possible to just write pages and pages of content, but I find it then becomes difficult, weeks or months down the line, to get a sense of the timeline of your previous work, what you were doing, and why you were doing it. Again, you don't have to use this system (no one will see your notebook besides you), but it is good to have some system you understand to make later reference easier.

## References

- [1] P. C. Brown, H. L. Roediger, and M. A. McDaniel, *Make it stick*. Harvard University Press, 2014.
- [2] K. P. Lee, "A guide to writing mathematics."
- [3] F. E. Su, "Writing mathematics well,"