Pseudo-Goldstino to Gravitino Decay: An Implication of Multiple Supersymmetry Breaking

by

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Abstract

This thesis studies the decay of a pseudo-goldstino to a gravitino plus a photon in the Minimal Supersymmetric Standard Model. The foundational premise of this decay process is that there are two independent sectors of supersymmetry breaking. We compute this main decay rate using the goldstino equivalence theorem to replace the final gravitino state with a goldstino. This replacement allows us to study simpler models which help build the intuition and methods for the final calculation. Specifically, we first study the decay of a pseudo-goldstino to a goldstino plus a photon in a toy model of multiple supersymmetry breaking and then the same process in the Minimal Supersymmetric Standard Model without supergravity. Incorporating supergravity introduces the interpretation of the goldstino as the longitudinal component of the gravitino and introduces the constant mass ratio between the gravitino and the pseudo-goldstino which is definitive of multiple local supersymmetry breaking. For the main decay process, we find that the rate is zero for certain relationships between the parameters which define the two hidden sectors. In the discussion we suggest other similar calculations which can be done within the same framework.

Thesis Supervisor: Jesse D. Thaler

Title: Assistant Professor of Physics

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

The standard model is currently the best tested comprehensive model of particle interactions. It contains theoretical descriptions of how the forces responsible for nuclear energy, radioactive decay, and electromagnetism are related to one another and how all currently observed particles interact through these forces. However, there are theoretical and experimental motivations for believing that the standard model is just an effective theory for a more encompassing theory.

From a theoretical perspective, the hierarchy problem is considered a major inconsistency in the standard model. The hierarchy problem refers to the huge order of magnitude disparity ("hierarchy") between the weak scale, which is well described by the quantum field theory of the standard model, and the Planck scale, at which quantum field theory is expected to break down due to fluctuations in spacetime. This disparity poses a problem to the standard model because it suggests that certain combinations of parameters must be finely tuned to 30 decimal places [1], an unlikely possibility.

From an experimental standpoint, there are many suggestions that there is a phenomenological theory of particle interactions which exists beyond the standard model. Neutrino masses, for example, are predicted to be zero in the renormalizable Standard model, but the observations of and models associated with neutrino oscillations suggest otherwise [2]. The standard model also says nothing about the asymmetry in the occurrence/existence of matter and antimatter in our physical world. Dark energy, the hypothetical and nebulous energy which permeates all of space, is possibly related to the vacuum energy which arises in all quantum field theories but the explicit predictions in the standard model which attempt to connect the theoretical vacuum energy to an experimental quantity miss the mark by 120 orders of magnitude [3]. Finally, dark matter a substance which is named for its apparent lack of interaction with electromagnetic radiation is responsible for most of the matter of the universe but there are no candidate standard model particles which satisfy its currently known properties [4].

With all of these motivations, we can state rather definitively that the standard model is incomplete from both an aesthetic and an experimental perspective. From here we can, and many people have, proceed in many directions to determine what models of physics, based on different constraining principles, exist beyond the standard model [5], but for this thesis will focus on the supersymmetric extension of the standard model.

Supersymmetry (termed 'SUSY' for short) claims that for each currently observed particle there is an as of yet undiscovered partner particle - called the superpartner. The particle and its superpartner have spins which differ by a half integer, so that SUSY essentially claims that all particles exist in boson/fermion pairs. A theory which is supersymmetric allows the exchange of certain bosons and fermions without changing the physical properties of the theory akin to the way a rotationally symmetric figure can be rotated without changing shape. It must be noted that although incorporating SUSY is a popular way to extend the standard model, it does not so much as solve all of the problems of the standard as much as it provides more room in which to build models to encompass the theoretical phenomenological inconsistencies [6].

If supersymmetry is a physical symmetry of nature, it must be a broken symmetry because observable particles do not come in mass degenerate boson/fermion pairs. Physicists state that a symmetry is broken in a system if the symmetry is mathematically present in the underlying theoretical structure of the system but is not present in the physical properties of the system. For, example we say that a ferromagnet has a broken rotational symmetry because although the physical laws governing the properties of the ferromagnet (i.e., Maxwell's equations) are rotationally invariant, the directional magnetic field in the ferromagnet is not. SUSY is a broken symmetry because although observed particles do not exist in boson/fermion, the mathematical structure of SUSY assumes they do.

Since supersymmetry (if it is physically manifest) must be broken, an important task for the model builder in creating a realistic model of supersymmetric particle interactions is to determine the mechanism of SUSY breaking. In typical models of SUSY breaking

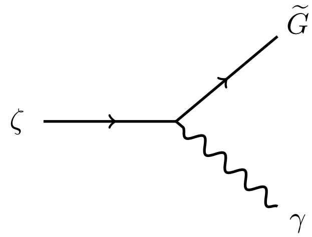

Figure 1-1: Pseudo-Goldstino to Gravitino Decay through MDI: When we have two sectors of breaking in supergravity, the pseudo-goldstino becomes the next-to-lightest supersymmetric particle allowing for the decay from a pseudo-goldstino to a gravitino via a magnetic dipole interaction.

one assumes there is a hidden sector of particles distinct from the observable particles and their superpartners. This hidden sector then induces supersymmetry breaking which is communicated to the visible sector through a messenger sector [7]. This thesis departs from the typical models of SUSY breaking by generalizing the assumption of a single sector of breaking to two sectors of breaking, a framework first proposed in [8]. A specific consequence of such a framework is that a small mass fermion, the pseudo-goldstino, is added to the low energy spectrum of the system. This addition allows for new phenomenological decay and scattering scenarios, one of which is investigated in this thesis. Specifically we compute the the decay rate for a pseudo-goldstino to go to a gravitino, the superpartner of the graviton, plus a photon through a magnetic dipole interaction (MDI) as shown in Fig. 1-1.

The outline of this thesis is as follows. In chapter 2, we provide a review of the supersymmetry background which is necessary to understand the later results of the thesis. In chapter 3, we develop the theoretical structure necessary for the main thesis problem by studying the main problem without supergravity (SUGRA). In chapter 4, we include SUGRA and derive the couplings and masses which are necessary for a computation of the pseudo-goldstino to gravitino plus photon decay rate. In chapter 5, we discuss ways to extend the study and consider ways to apply the result to models of dark matter.

Chapter 2

Supersymmetry

Supersymmetry was "discovered" close to 40 years ago [9] and since then physicists have progressed a long way in the development of the theoretical and phenomenological structure of the subject. In this chapter we provide a short review of the developments which exist as a foundation to the results in this thesis.

In the first section we outline the basics of supersymmetry expressed in terms of the component fields of the simplest multiplets. In the second section we translate these supersymmetry basics to the superfield formalism. In the third section we discuss supersymmetry breaking and lay the foundation for the scenarios of multiple breaking discussed in the next chapter. In the final section we summarize the Minimal Supersymmetric Standard Model and derive the 4×4 neutralino mass matrix (without soft supersymmetry breaking terms) which is largely relevant to our later analysis.

2.1 Basics

There are two routes towards the construction of a supersymmetric theory. One can proceed intuitively by choosing an appropriate collection of boson and fermion fields and incorporating the supersymmetric criterion by hand, or one can proceed formally by considering supersymmetric algebras and then constructing superfields which fall into various representations of the algebra. Here we will eschew formality and take the intuitive approach and later substantiate it with the more powerful results of the superfield formalism. The analysis in the subsequent sections will very much follow the discussion in [7].

2.1.1 Wess-Zumino Models

In order to construct a supersymmetric theory we need to find a transformation which transforms bosons into fermions and vice versa. The simplest action containing both boson and fermion degrees of freedom is

$$S = \int d^4x \, (\partial^\mu \phi \partial_\mu \phi^* + i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi), \tag{2.1.1}$$

where ϕ is a complex scalar field and ψ is a Weyl field. A set of transformations which leaves the action invariant while exchanging boson and fermion degrees of freedom is

$$\delta_\epsilon \phi = \epsilon^\alpha \psi_\alpha \equiv \epsilon\psi, \tag{2.1.2}$$

$$\delta_\epsilon \psi_\alpha = -(\sigma^\nu \epsilon^\dagger)_\alpha \partial_\nu \phi, \tag{2.1.3}$$

where ϵ is a infinitesimal constant spinor with mass dimension $-1/2$. If these transformations are to be mathematically consistent, however, we must ensure that they close. Colloquially, we must ensure that applying a series of transformations does not result in a net transformation which does not represent a symmetry of the theory. We ensure closure by calculating a commutator of two transformations and inspecting the result. The commutator of the bosonic field transformation yields

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\phi. \tag{2.1.4}$$

We see that instead of obtaining another SUSY transformation we have a partial derivative. The partial derivative is the generator of spacetime translations so the fact that it results from the commutation of two SUSY transformations suggests that the SUSY algebra contains the translational spacetime symmetry too. This fact is important in establishing supersymmetry as an extension to the already known spacetime symmetries. Computing the commutator of the transformations for the fermionic field we have

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha = -i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\partial_\mu\psi_\alpha \tag{2.1.5}$$

$$+ i(\epsilon_{1\alpha}\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - \epsilon_{2\alpha}\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi). \tag{2.1.6}$$

If we allow the fermions to be on shell, the last term vanishes and we have a result analogous to the boson result above. However, if we take the fermions to be off shell then the additional term seems to rule out our desired closure property. The solution to this problem is to recognize that we cannot achieve closure off-shell with our current field content, but must introduce an additional field. The field to introduce turns out to be an auxiliary field of mass dimension 2 without field dynamics of its own. Its lagrangian and transformation are respectively

$$\mathcal{L}_F = F^\dagger F, \tag{2.1.7}$$

$$\delta_\epsilon F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi. \tag{2.1.8}$$

A fortunate bonus is that the transformations of F are already closed. To ensure that the action stays invariant with the inclusion of the auxiliary field, we must modify the transformation of the fermion field

$$\delta\psi_\alpha = -i(\sigma^\nu \epsilon^\dagger)_\alpha \partial_\nu \phi + \epsilon_\alpha F. \tag{2.1.9}$$

We note that the new term in the fermion transformation is proportional to the auxiliary field. This fact will be important when we discuss supersymmetry breaking. Now, the total lagrangian

$$\mathcal{L} = \partial^\mu \phi \partial_\mu \phi^* + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F, \tag{2.1.10}$$

is invariant under supersymmetry transformations, and the commutators of each transformation obeys

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = -i(\epsilon_1 \sigma^\mu \epsilon_2 - \epsilon_2 \sigma^\mu \epsilon_1)\partial_\mu X, \tag{2.1.11}$$

where X is any of the fields ϕ, ψ, F or their Hermitian conjugates.

We now have our simplest supersymmetric theory, but it is not very interesting because it does not contain any interactions. Building up the interactions to be consistent with supersymmetry requires some work, but what we ultimately find is that to maintain the supersymmetric invariance of the lagrangian we must have

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} W^{(2)} \psi\psi + W^{(1)} F + \text{h.c.}, \tag{2.1.12}$$

where

$$W = h\phi + \frac{1}{2}m\phi^2 + \frac{1}{3!}f\phi^3, \tag{2.1.13}$$

and

$$W^{(2)} = \frac{\partial^2}{\partial\phi^2}W, \qquad W^{(1)} = \frac{\partial}{\partial\phi}W \tag{2.1.14}$$

and h, m, and f are positive constants. We collect these results into a full interacting lagrangian for our boson ϕ and fermion ψ

$$\mathcal{L}_{\mathrm{WZ}} = \partial^\mu\phi\partial_\mu\phi^* + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi + F^\dagger F \tag{2.1.15}$$

$$- \frac{1}{2}W^{(2)}\psi\psi + W^{(1)}F + \text{h.c.}\,. \tag{2.1.16}$$

To obtain a lagrangian which only contains the physical fields we solve for the equations of motion of the auxiliary field F. Our current lagrangian only contains a single set of fields, but can be easily generalized to include an arbitrary number of fields. The fields which make up the above lagrangian define what is called a chiral multiplet, and the associated lagrangians are typically termed Wess-Zumino lagrangians.

2.1.2 Yang Mills Theories

Above we constructed a supersymmetric lagrangian with a spin 0 boson and spin 1/2 fermion. The next logical step is to construct a lagrangian with a spin 1 boson and its appropriate superpartner. Theories of this nature are in general termed SUSY Yang Mills theories since the most general lagrangian which represents the free dynamics of spin-1 particles is the Yang-Mills lagrangian. The construction of the supersymmetric Yang-Mills lagrangian proceeds analogously to the construction of the Wess-Zumino lagrangian so we merely state the result. The lagrangian is

$$\mathcal{L}_{SYM} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^a D^a, \tag{2.1.17}$$

where a is the generator label and D^a is an auxiliary field of mass dimension 2, and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu\,, \tag{2.1.18}$$

$$D_\mu\lambda^a = \partial_\mu\lambda^a - g f^{abc} A^b_\mu \lambda^c, \tag{2.1.19}$$

with λ is a Weyl fermion in general termed the gaugino and $A^{\mu a}$ is a non-abelian gauge field. The fields which make up this lagrangian define what is typically called a vector multiplet. The supersymmetry transformations of these fields are

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}[\epsilon^\dagger \bar{\sigma}_\mu \lambda^a + \lambda^{\dagger\, a} \bar{\sigma}_\mu \epsilon], \tag{2.1.20}$$

$$\delta \lambda^a_\alpha = -\frac{i}{2\sqrt{2}}(\sigma^\mu \bar{\sigma}^\nu \epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a, \tag{2.1.21}$$

$$\delta D^a = -\frac{i}{\sqrt{2}}[\epsilon^\dagger \bar{\sigma}^\mu D_\mu \lambda^a - D_\mu \lambda^{\dagger\, a} \bar{\sigma}^\mu \epsilon]. \tag{2.1.22}$$

2.1.3 Gauge Theories

Our principle concern in this thesis will be with supersymmetric gauge theories, that is theories which contain spin 0 bosons and gauge bosons in addition to their respective supersymmetric partners. In such theories the boson and the other fields in the chiral multiplet must transform under the gauge symmetry as

$$\delta_{\text{gauge}} X = ig\Lambda^a T^a X. \tag{2.1.23}$$

We can make our chiral multiplet lagrangian gauge invariant by promoting regular the spacetime derivatives in (2.1.10) to covariant derivatives

$$D_\mu \phi = \partial_\mu \phi + igA^a_\mu T^a \phi, \tag{2.1.24}$$

$$D_\mu \psi = \partial_\mu \psi + igA^a_\mu T^a \psi. \tag{2.1.25}$$

These covariant derivatives introduce interactions between the gauge boson and the members of the chiral multiplet. The requirements of supersymmetry suggest then that the gaugino λ and the auxiliary field D should in someway couple to these members too. When we introduce these couplings our gauge theory lagrangiran becomes

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{\text{WZ}} - \sqrt{2}g\left[(\phi^* T^a \psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a \phi)\right] + g(\phi^* T^a \phi)D^a, \tag{2.1.26}$$

where $\mathcal{L}_{\text{WZ}}$ is the Wess-Zumino lagrangian (2.1.16) with space time derivatives made covariant and $\mathcal{L}_{\text{SYM}}$ is the Super Yang Mills lagrangian (2.1.17). Clearly with these new interactions our previous supersymmetry transformations would most likely be modified.

Fortunately, the only modifications occur in the chiral multiplet. The new transformations for fields in the chiral multiplet are

$$\delta\phi = \epsilon\psi, \tag{2.1.27}$$

$$\delta_\psi = -i(\sigma^\mu\epsilon^\dagger)_\alpha D_\mu\phi + \epsilon_\alpha F, \tag{2.1.28}$$

$$\delta F = -i\epsilon^\dagger\bar{\sigma}^\mu D_\mu\psi + \sqrt{2}g(T^a\phi)\epsilon^\dagger\lambda^{\dagger\, a}, \tag{2.1.29}$$

and all other transformations remain the same. These transformations leave the action for (2.1.26) invariant under supersymmetry transformations if we mandate that the $W(\phi)$ is gauge invariant. Namely

$$\delta_{\text{gauge}}W = ig\Lambda^a\frac{\partial W}{\partial\phi}T^a\phi = 0. \tag{2.1.30}$$

2.2 Superspace Formalism

When we have systems involving many different supersymmetry multiplets it is best to adopt the superspace formalism and collect fields in the same multiplet into what is known as a superfield. In this section we review this formalism closely following the review in [7]. In the superspace formalism we add two two-component Grassmann coordinates to the four spacetime coordinates. These coordinates are termed spinors and they satisfy the anticommuting relation

$$\{\theta_\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0. \tag{2.2.1}$$

We can define integration over these coordinates using the definition of Grassmann integration

$$\int d\eta = 0, \quad \int \eta d\eta = 1, \tag{2.2.2}$$

and we can define the integration differentials as

$$d^2\theta = -\frac{1}{4}d\theta^\alpha d\theta^\beta\epsilon_{\alpha\beta}, \tag{2.2.3}$$

$$d^2\bar{\theta} = -\frac{1}{4}d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}_{\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}}, \tag{2.2.4}$$

$$d^4\theta = d^2\theta d^2\bar{\theta}, \tag{2.2.5}$$

where $\epsilon^{\alpha\beta}$ is the two component levi civita symbol. General spinor identities for the θ coordinates can be found in Section 3.2 of [10] and the appendix of [7].

We collect the components of the particle multiplets mentioned in the last section into what is known as superfields. Superfields are most easily constructed as a function of the superspace coordinate y where

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}. \tag{2.2.6}$$

For example the superfield for the chiral multiplet, known as the chiral superfield, is

$$\begin{aligned}\boldsymbol{\Phi}(y) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) & (2.2.7)\\ &= \phi(x) - i\theta\sigma^\mu\bar{\theta}\,\partial_\mu\phi(x) - \frac{1}{4}\theta^2\bar{\theta}^2\,\partial^2\phi(x) & (2.2.8)\\ &\quad + \sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}\theta^2\,\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta^2 F(x) & (2.2.9)\end{aligned}$$

where the second line is obtained by Taylor expanding the first line and using spinor identities. In this thesis we will write superfields in bold type to distinguish them from their component fields. Using the spinor identities we can show that our Wess-Zumino lagrangian (2.1.16) is reproduced by the lagrangian

$$\mathcal{L}_{\mathrm{WZ}} = \int d^4\theta\,\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi} + \left(\int d^2\theta\,W(\boldsymbol{\Phi}) + \mathrm{h.c.}\right), \tag{2.2.10}$$

where

$$W(\boldsymbol{\Phi}) = h\Phi + \frac{1}{2}\mu\boldsymbol{\Phi}^2 + \frac{1}{3!}f\boldsymbol{\Phi}^3. \tag{2.2.11}$$

The second term term in (2.2.10) is typically called the superpotential. The first term is called the Kähler potential and it is often written functionally as $K(\boldsymbol{\Phi}^\dagger,\boldsymbol{\Phi}) = \boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}$.

We express our vector multiplet in terms of what is known as a vector superfield

$$\boldsymbol{V}^a(x) = \theta\bar{\sigma}^\mu\bar{\theta}\,A^a_\mu + i\theta^2\bar{\theta}\lambda^{\dagger\,a} - i\theta\bar{\theta}^2\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2\,D^a. \tag{2.2.12}$$

In the above expression the vector superfield is written in what is known as *Wess-Zumino gauge* where the extra spinors and scalar fields which usually make up the vector superfield have been removed by a gauge transformation. The gauge transformation of the vector superfield is defined by

$$e^{T^a\boldsymbol{V}^a} \rightarrow e^{T^a\boldsymbol{\Lambda}^{a\,\dagger}}e^{T^a\boldsymbol{V}^a}e^{T^a\boldsymbol{\Lambda}^a}, \tag{2.2.13}$$

or to a perturbative order by

$$\boldsymbol{V}^a \to \boldsymbol{V}^a + \boldsymbol{\Lambda}^a + \boldsymbol{\Lambda}^{a\,\dagger} + \mathcal{O}(\boldsymbol{V}^a\boldsymbol{\Lambda}^a), \tag{2.2.14}$$

where T^a are the generators of the gauge group and $\boldsymbol{\Lambda}^a$ are a collection of chiral superfields.

In order to build the kinetic terms for this vector superfield which reproduce the supersymmetric Yang-Mills lagrangian (2.1.17), we must define the vector superfield strength

$$T^a \boldsymbol{W}^a_\alpha \equiv -\frac{1}{4}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}} e^{-T^a\boldsymbol{V}^a} D_\alpha e^{T^a\boldsymbol{V}^a}, \tag{2.2.15}$$

where

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - 2i(\sigma^\mu\bar{\theta})_\alpha \frac{\partial}{\partial y^\mu}, \tag{2.2.16}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}, \tag{2.2.17}$$

are superspace derivatives written in y space. Applying the derivatives and expanding (2.2.15) and applying spinor identities yields

$$\boldsymbol{W}^a_\alpha = -i\lambda^a_\alpha(y) + \theta_\alpha D^a(y) - (\sigma^{\mu\nu}\theta)_\alpha F^a_{\mu\nu}(y) - \theta\,\theta\sigma^\mu D_\mu \lambda^{\dagger\,a}(y), \tag{2.2.18}$$

where $\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$. The supersymmetric Yang Mills Lagrangian can then be written as

$$\mathcal{L}_{\text{SYM}} = \int d^2\theta\, \frac{1}{4}\boldsymbol{W}^{a\,\alpha}\boldsymbol{W}^a_\alpha + \text{h.c.} \tag{2.2.19}$$

To construct supersymmetric gauge theories in superspace, we must couple the chiral superfield to the vector superfield in a gauge-invariant way. The structure of the vector superfield gauge transformation in (2.2.13) gives us a clue as to what the coupling should be. When going from a non-gauged to a gauged theory the Kähler potential becomes

$$K(\boldsymbol{\Phi}^\dagger, \boldsymbol{\Phi}) = \boldsymbol{\Phi}^\dagger e^{2gT^aV^a}\boldsymbol{\Phi}. \tag{2.2.20}$$

Also in order to ensure this Kähler potential is gauge invariant, we require $\boldsymbol{\Phi}$ to transform as

$$\boldsymbol{\Phi} \to e^{-gT^a\boldsymbol{\Lambda}^a}\boldsymbol{\Phi}. \tag{2.2.21}$$

In summary, the general lagrangian for a renormalizable supersymmetric gauge theory can be written in superfield formalism as

$$\mathcal{L}_{\text{SUSY}} = \int d^4\theta\, \boldsymbol{\Phi}^\dagger e^{2gT^aV^a} \boldsymbol{\Phi} + \left(\int d^2\theta\, W(\boldsymbol{\Phi}) + \frac{1}{4} \boldsymbol{W}^{a\,\alpha} \boldsymbol{W}^a_\alpha + \text{h.c.} \right). \tag{2.2.22}$$

This lagrangian reproduces (2.1.26) when expanded in component fields and integrated over superspace.

2.3 Supersymmetry Breaking

If supersymmetry is a physical symmetry of nature, the fact that it is not immediately manifest in the current spectrum of particles tells us quite definitely that it is a broken symmetry. Exactly how supersymmetry is broken, however, is much less definite. In this section we review the standard mechanisms for supersymmetry breaking and conclude with a comment on how the premise of this thesis modifies these mechanisms.

2.3.1 Spontaneous Symmetry Breaking

If a symmetry is not manifest in the physical properties of the system, but the symmetry is present in the mathematical representation of the system, then we say the symmetry is spontaneously broken. For example, the following lagrangian models the interactions between a massless field $b(x)$ and a massive field $a(x)$

$$\mathcal{L} = \frac{1}{2}\partial^\mu a \partial_\mu a + \frac{1}{2}\partial^\mu b \partial_\mu b \tag{2.3.1}$$

$$- \mu^2 a^2 - \frac{\sqrt{\lambda}}{2}\mu a(a^2 + b^2) - \frac{\lambda}{16}(a^2 + b^2)^2. \tag{2.3.2}$$

From a naive inspection of the above lagrangian we would conclude there was no symmetry which related the two fields to one another. However if we reparameterize our fields as

$$\phi(x) = \frac{1}{2}\left[v + a(x) + ib(x)\right], \tag{2.3.3}$$

where $v = 2m/\sqrt{\lambda}$ we find that we can rewrite (2.3.2) as

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - \mu^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2. \tag{2.3.4}$$

In this form we realize that the original lagrangian had a U(1) (or equivalently an SO(2)) symmetry. If we were to study these two fields through collisions, it would have been difficult to realize that there was a symmetry relating them even though the symmetry is present in the lagrangian (2.3.2) which defines their interactions. The main lesson to draw from this example is that a symmetry can be present in the mathematical structure of a system, even if it is not present in the physical properties of that system. This can occur if the symmetry is spontaneously broken. Extrapolating this argument to supersymmetry, we can claim that supersymmetry - if it is a legitimate symmetry - must be spontaneously broken.

One feature which hails the existence of a spontaneously broken symmetry is the existence of a massless particle. For bosonic symmetries this massless particle is called the goldstone boson. In the example above the goldstone boson was the field $b(x)$. In supersymmetric theories, in which the defining symmetry group is fermionic, the massless particle is a fermion called the goldstino.

Since the goldstino factors largely in our later analysis we will take some time to review one of its important properties. In a general renormalizable supersymmetric theory with both vector and chiral field multiplets, the mass bilinear fermion terms are

$$\mathcal{L}_{FM} = -\frac{1}{2}W^{jk}\psi_j\psi_k - \sqrt{2}g(\phi^* T^a \psi)\lambda^a + \text{h.c.}\,, \tag{2.3.5}$$

where W^{jk} is the second derivative of the superpotential, ψ_j is a fermion in the chiral multiple j, g is the gauge coupling for a gauge group with generators T^a, and λ^a are the gauginos in the vector multiplet. Taking the coefficients of the bilinear fermion terms to be evaluated at their VEVs we can state the mass matrix of fermions in a general renormalizable SUSY theory to be

$$M_{\text{Fermion}} = \begin{pmatrix} 0 & \sqrt{2}g(\langle\phi^*\rangle T^a)^i \\ \sqrt{2}g(\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix}, \tag{2.3.6}$$

where the matrix is written in the (λ^a, ψ_i) basis. If supersymmetry is broken, this matrix has a zero eiegenvector corresponding to the godlstino state. The eiegenvector is

$$|\eta\rangle = \frac{1}{F_\eta} \begin{pmatrix} \langle D^a \rangle / \sqrt{2} \\ \langle F_i \rangle \end{pmatrix}, \tag{2.3.7}$$

where $F_\eta = \sum_i \langle F_i \rangle^2 + \sum_a \langle D^a \rangle^2 / 2$, and once again we have chosen the (λ^a, ψ_i) basis. It can be shown that this eigenvector yields zero when applied to the fermion mass matrix by using two facts: one, that the superpotential is invariant under gauge transformations (2.1.30); and two, that the first derivative boson potential has a zero VEV.

2.3.2 Nonlinear Parameterization of Chiral Superfield

Here we will derive a useful parameterization of supersymmetry breaking chiral superfields which isolates the dynamics of the goldsitno [11]. We begin by considering global symmetry breaking in a simpler theory. We know that in a broken SU(N) theory, the goldstone fields π^a are included in the complex scalar field ϕ^a via the parameterization

$$\phi = \exp(i\pi^A(x) t^A / v) \langle \phi \rangle, \tag{2.3.8}$$

where $\langle \phi \rangle$ is the VEV of ϕ, and t^A are the broken generators of the gauge group SU(N). This particular parameterization of ϕ is useful for studying the low energy implication of symmetry breaking because it only contains the massless goldstone fields.

In direct analogy to the above parameterization of goldstone fields, we can study the low energy dynamics of a theory with supersymmetry breaking by writing the symmetry breaking chiral superfield as

$$\boldsymbol{X} = \exp\left(\eta(x) Q / \sqrt{2} F_X + \overline{\eta}(x) \overline{Q} / \sqrt{2} F_X\right) \langle \boldsymbol{X} \rangle, \tag{2.3.9}$$

where $\langle \boldsymbol{X} \rangle = \theta^2 \langle F_X \rangle$ is the VEV of $\boldsymbol{X}$, η is the goldstino field, and Q is the generator of SUSY transformations. Since F_X is a constant and $\langle \boldsymbol{X} \rangle$ does not contain $\bar{\theta}$, $\boldsymbol{X}$ reduces to

$$\begin{aligned}
\boldsymbol{X} &= \exp\left(\eta(x)\frac{\partial}{\partial\theta}/\sqrt{2}F_X\right)\langle \boldsymbol{X}\rangle \\
&= \left(1 + \frac{\eta}{\sqrt{2}\langle F_X\rangle}\frac{\partial}{\partial\theta} + \frac{1}{2}\frac{\eta\eta}{2\langle F\rangle^2}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta}\right)\theta^2\langle F_X\rangle \\
&= \theta^2\langle F_X\rangle + \sqrt{2}\eta\,\theta + \frac{\eta\eta}{2\langle F_X\rangle}. \qquad (2.3.10)
\end{aligned}$$

We term (2.3.10) the nonlinear parameterization. This parameterization is useful because it allows us to focus on the goldstino degree of freedom whenever we break supersymmetry. We note that we could have derived this parameterization by enforcing the massless condition $\boldsymbol{X}^2 = 0$ and solving for the scalar component of $\boldsymbol{X}$. From now on, whenever we write $\boldsymbol{X}$ we are referring to the nonlinear parameterization in (2.3.10).

2.3.3 Soft-SUSY Breaking

In addition to breaking supersymmetry spontaneously, we should include the possibility for explicit breaking. In fact for the supersymmetric standard model there are phenomenological reasons why explicit breaking is preferred over spontaneous breaking [10]. In explicit supersymmetry breaking, symmetry violating terms are added to a supersymmetry preserving lagrangian to produce

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{Soft}}. \qquad (2.3.11)$$

where the symmetry violating terms $\mathcal{L}_{\text{Soft}}$ are denoted “soft” by convention. Although these terms break supersymmetry, they must not violate the high energy convergence property of supersymmetric theories and cannot change the non-renormalization of the superpotential. It turns out these constraints are only satisfied if the terms which make up $\mathcal{L}_{\text{Soft}}$ have a mass dimension less than 4. We call such terms “soft” mass terms. For example, in a general renormalizable supersymmetric theory we have the lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{SUSY}} = &\int d^4\theta\, \boldsymbol{\Phi}_i^\dagger (e^{2g\boldsymbol{V}^a T^a})^i_j \boldsymbol{\Phi}^j + \left(\int d^2\theta\, \frac{1}{4}\boldsymbol{W}^{a\,\alpha}\boldsymbol{W}^a_\alpha + \text{h.c.}\right) \qquad (2.3.12) \\
&+ \left(\int d^2\theta\, h_i\boldsymbol{\Phi}_i + \frac{1}{2}\mu_{ij}\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j + \frac{1}{3!}\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j\boldsymbol{\Phi}_j + \text{h.c.}\right) \qquad (2.3.13)
\end{aligned}$$

and the most general soft-supersymmetry breaking terms which can be added to this lagrangian are [12]

$$\mathcal{L}_{\text{Soft}} = -(\tilde{m}^2)^i_j \phi^\dagger_i \phi^j - \left(\frac{m_\lambda}{2} \lambda^a \lambda^a + C_i \phi_i + \frac{B_{ij}}{2} \phi_i \phi_j + \frac{A_{ijk}}{3!} \phi_i \phi_j \phi_k \right), \tag{2.3.14}$$

These soft-breaking terms are useful because they parameterize our ignorance of the mechanism responsible for supersymmetry breaking. However, in some situations it is useful to explicitly include the source of supersymmetry breaking through effective interactions. In the typical soft-supersymmetry breaking scenario, supersymmetry breaking is said to occur first in a hidden sector. We call the sector responsible for supersymmetry breaking "hidden" because it is a singlet under all the gauge groups of the visible sector and hence does not interact directly with (and is "hidden" from) the gauge sector fields. Through the interactions between the hidden sector and the visible sector, supersymmetry is broken in the visible sector thus producing the terms in (2.3.14). But it is possible to obtain (2.3.14) by expicitly writing the interaction lagrangian between the fields in the visible sector and a single nonlinearly parameterized superfield comprising the hidden sector. The correct lagrangian to reproduce the above soft supersymmetry breaking terms is [7]

$$\begin{aligned} \mathcal{L} = -\int d^4\theta \frac{\tilde{m}_i^2}{F_X^2} \boldsymbol{X}^\dagger \boldsymbol{X}\, \boldsymbol{\Phi}_i^\dagger (e^{2g\boldsymbol{V}^a T^a})^i_j \boldsymbol{\Phi}^j - \bigg(\int d^2\theta\, \frac{m_\lambda}{2F_X} \boldsymbol{X}\, \boldsymbol{W}^{\alpha\, a} \boldsymbol{W}^a_\alpha + \frac{C_i}{F_X} \boldsymbol{X} \boldsymbol{\Phi}_i \\ + \frac{B_{ij}}{2F_X} \boldsymbol{X} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_j + \frac{A_{ijk}}{3! F_X} \boldsymbol{X} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_j \boldsymbol{\Phi}_k + \text{h.c.} \bigg), \end{aligned} \tag{2.3.15}$$

where F_X is the VEV of the F component of X. We call the above terms effective soft-supersymmetry breaking interactions. In this lagrangian $\boldsymbol{X}$ is non-linearly parameterized according to (2.3.10) and taking $\boldsymbol{X}$ to be evaluated at its VEV we reproduce (2.3.14). When $\boldsymbol{X}$ is not at its VEV, the lagrangian produces interactions between the fermion of the hidden sector η and the visible sector component fields. We will consider the implications of a few of these interactions in a toy model in the next chapter.

2.4 The Standard Model and the MSSM

In this section we review the construction of the Minimal Supersymmetric Standard Model. Since there are many pedagogical sources (e.g., [13] [1] [10] [7]) which provide a more complete overview of the MSSM, our review will focus mostly on the results which are essential for the subsequent portions of this thesis.

2.4.1 The Standard Model

In constructing the MSSM, we begin first by reviewing the standard model. The standard model is the gauge theory which defines how the elementary particles interact with one another through the electromagnetic, weak, and strong interactions. Its defining gauge group is $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ where $\mathrm{SU}(3)_C$ is the gauge group for the strong interactions and $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is the group for the so called electroweak interactions. The non-gauge field content of the standard model is summarized in the Table 2.1. The first five rows are left-handed Weyl fields, the first two of which define the leptons and the next three of which define the quarks. Both leptons and quarks come in three copies, termed generations, in the standard model. The last row defines the Higgs field which is a complex scalar field. It is the Higgs field which is responsible for the breaking of the electroweak symmetry down to electromagnetism. These matter fields couple to the gauge fields according to their charges and their representations in the gauge group. What is important, then, with regard to the supersymmetry extrapolation of the standard model is the interactions between the matter fields. In the standard model these interactions are

$$\mathcal{L}_{\mathrm{Yuk}} = -H\cdot L_i Y_{ij}\overline{e}_j - H\cdot Q_i Y'_{ij}\overline{d}_j - H^\dagger\, Q_i Y''_{ij}\overline{u}^\alpha_j + \text{h.c.}\,, \tag{2.4.1}$$

$$V(H,H^\dagger) = -\frac{\lambda}{4}(H^\dagger H - \tfrac{1}{2}v^2)^2, \tag{2.4.2}$$

where i is a generation index and where $A\cdot B = \varepsilon_{\alpha\beta}A_\alpha B_\beta$ defines an SU(2) invariant product between A and B. With these interactions stated, we can now extrapolate to the supersymmetric case.

2.4.2 The MSSM

In this section we review the basics of the Minimal Supersymmetric Standard Model (MSSM). The “minimal” nature of this supersymmetric extension exists in the fact that it makes the least number of assumptions concerning what a supersymmetric standard model may be. Only fields which are absolutely necessary to the theoretical and phenomenological consis-

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$Q_i = (u_L, d_L)_i$	3	2	$+\frac{1}{6}$
$\overline{u}_i$	$\overline{3}$	1	$-\frac{2}{3}$
$\overline{d}_i$	$\overline{3}$	1	$+\frac{1}{3}$
$L_i = (\nu, e_L)_i$	1	2	$-\frac{1}{2}$
$\overline{e}_i$	1	1	$+1$
$H_i = (H^0, H^-)_i$	1	2	$-\frac{1}{2}$

Table 2.1: Particle content of the standard model. All matter fields are fermions except for the higgs field.

tency of a supersymmetric standard model are added into the picture.

In determining a supersymmetric extension of the standard model, we proceed with the basic assumption that none of the currently observed particles are superpartners of each other. This fact seems obvious in retrospect but historically was not always clear [14]. With this assumption we can then naively construct our supersymmetric extension by placing all of the observed particles in appropriate multiplets. The higgs field, since it contains dynamic scalar degrees of freedom, can only be part of a chiral multiplet. Since the leptons and quarks are in the fundamental or singlet representation (as opposed to the adjoint representation) of the gauge group, they also can only be part of chiral multiplets (as opposed to vector multiplets). The gauge fields clearly can only be part of vector multiplets. By placing the original standard model fields in particular multiplets we effectively double the particle content of our theory. With this doubling for each standard model particle there is a boson or fermion superpartner with identical quantum numbers to the original particle. When writing a supersymmetric standard model we replace the original fields with their chiral or vector superfields but retain the standard model name of the field.

From here we could proceed with the construction of a supersymmetric standard model lagrangian, but we would run into a theoretical inconsistency due to the new fermions in our theory. All of the potential anomalies of the standard model which would violate the gauge symmetries at a quantum level are conveniently canceled by the very choice of quantum numbers which make sense phenomenologically. When we consider a supersymmetric standard model, we introduce new fermions (gauginos and higgsino) which can lead to newanomalies. Since gauginos couple vectorially, they do not contribute to the chiral anomalies, but the higgsinos does introduce anomalies. In a triangle diagram with three $\mathrm{U}(1)_Y$ gauge fields, the higgsino loop would be proportional to $Y^3 = (-1)^3$ which is nonzero and hence anomaly

	bosons	fermions	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_i	$(\widetilde{u}_L, \widetilde{d}_L)_i$	$(u_L, d_L)_i$	3	2	$\frac{1}{6}$
$\overline{u}_i$	$\widetilde{u}^*_{Ri}$	$\overline{u}_i = u^\dagger_{Ri}$	$\overline{3}$	1	$-\frac{2}{3}$
$\overline{d}_i$	$\widetilde{d}^*_{Ri}$	$\overline{d}_i = d^\dagger_{Ri}$	$\overline{3}$	1	$\frac{1}{3}$
L_i	$(\widetilde{\nu}, \widetilde{e}_L)_i$	$(\nu, e_L)_i$	1	2	$-\frac{1}{2}$
$\overline{e}_i$	$\widetilde{e}^*_{Ri}$	$\overline{e}_i = e^\dagger_{Ri}$	1	2	1
H_u	(H^+_u, H^0_u)	$(\widetilde{H}^+_u, \widetilde{H}^0_u)$	1	2	$\frac{1}{2}$
H_d	(H^0_u, H^-_u)	$(\widetilde{H}^0_u, \widetilde{H}^-_u)$	1	2	-$\frac{1}{2}$

Table 2.2: Matter field particle content of the MSSM.

inducing. This problem is fixed by introducing a second higgs doublet with opposite hypercharge, so that the coefficient of the loop becomes $Y^3 = (-1)^3 + (+1)^3 = 0$ [14]. To differentiate between the two higgs doublets (and in anticipation of how each higgs couples to the quarks) we label the higgs doublet with positive hypercharge with a u and the higgs doublet with negative hypercharge with a d. We can now tabulate our particle spectrum as we have done in Table 2.2. In the MSSM, the structure of the canonical kinetic terms is completely determined by the quantum numbers and group representations listed in the above table. Therefore the dynamics of the MSSM and the theories which extend from it is determined by the superpotential or interaction lagrangians in general. In writing down the MSSM superpotential we can simply take the Yukawa interactions of the standard model and label the fields as chiral superfields. Doing so gives us the superpotential

$$W = \overline{\boldsymbol{u}} Y_u \boldsymbol{Q} \cdot \boldsymbol{H}_u - \overline{\boldsymbol{d}} Y_d \boldsymbol{Q} \cdot \boldsymbol{H}_d - \overline{\boldsymbol{e}} Y_e \boldsymbol{L} \cdot \boldsymbol{H}_d \tag{2.4.3}$$

where we replaced the complex conjugate of the original higgs field with the newly introduced higgs doublet. However, with the above superpotential we have no massive higgs excitation. This problem is fixed by including a gauge invariant product of our two higgs doublets to the superpotential:

$$W_{\text{Higgs}} = \mu \boldsymbol{H}_u \cdot \boldsymbol{H}_d. \tag{2.4.4}$$

There are additional terms which can be included in this superpotential but many of these terms violate lepton or baryon number. The standard approach to excluding these terms is to postulate a new symmetry called $R-$parity [14].

2.4.3 Neutralino Mass Matrix

Later we will want to compute the mass matrix for neutral fermions in the MSSM coupled to two hidden sectors. As a precursor to this calculation, we should calculate bare mass matrix for the neutral fermions in the MSSM alone. We begin with the general mass matrix for fermions in a renormalizable supersymmetric theory (2.3.6), reproduced here for convenience:

$$M_{\text{Fermion}} = \begin{pmatrix} 0 & \sqrt{2}g(\langle\phi^*\rangle T^a)^i \\ \sqrt{2}g(\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix}. \tag{2.4.5}$$

From (2.4.5) it is clear the calculation of the MSSM neutral fermion mass matrix will require an accounting of gauge group generators and the superpotential terms. In the MSSM we only take the up and down chiral superfields to have VEVs. Moreover only the hypercharge gauge group and the third "direction" of the SU(2) group will yield neutral gauginos. Also, only one component of each higgs doublet will yield a neutral fermion so we focus on that component. With Y and T^3 defined as generators of hypercharge and the third-direction of the SU(2) group, respectively, we have the following relations

$$Y\,H_u^0 = \frac{1}{2}H_u^0, \qquad T^3\,H_u^0 = -\frac{1}{2}H_u^0, \tag{2.4.6}$$

$$Y\,H_d^0 = -\frac{1}{2}H_d^0, \qquad T^3\,H_d^0 = \frac{1}{2}H_d^0, \tag{2.4.7}$$

where

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^0 \\ H_u^- \end{pmatrix}. \tag{2.4.8}$$

With the definition $\langle H_u^0\rangle = v_u/\sqrt{2}$ and $\langle H_d^0\rangle = v_d/\sqrt{2}$, the gauge part of the neutralino mass matrix is complete. For the superpotential part, we need only focus on the terms with higgs fields, namely

$$W_{\text{Higgs}} = \mu\boldsymbol{H}_u \cdot \boldsymbol{H}_d = \mu(\boldsymbol{H}_u^+\boldsymbol{H}_d^- - \boldsymbol{H}_u^0\boldsymbol{H}_d^0). \tag{2.4.9}$$

From this expression, and our previous consideration of how the gauge group generators act on the higgs vevs, we can easily write down the neutralino mass matrix in the non-broken MSSM:

$$M_{\tilde{N}^0}^{4\times 4} = \begin{pmatrix} 0 & 0 & g_Y v_u/\sqrt{2} & -g_Y v_d/\sqrt{2} \\ 0 & 0 & -g v_u/\sqrt{2} & g v_d/\sqrt{2} \\ g_Y v_u/\sqrt{2} & -g v_u/\sqrt{2} & 0 & -\mu \\ -g_Y v_d/\sqrt{2} & g v_d/\sqrt{2} & -\mu & 0 \end{pmatrix}, \tag{2.4.10}$$

where we have chosen the $(\tilde{B}, \tilde{W}^3, \tilde{H}_u^0, \tilde{H}_d^0)$ basis for the matrix. Now that we have complete a computation of the neutralino mass matrix in the MSSM without breaking, we can now consider how our result is modified by the introduction of one or multiple symmetry breaking fields. We will do so in the next chapter.

2.4.4 Soft SUSY Breaking in the MSSM

We have discussed the supersymmetric extension of the standard model but we have yet to discuss how supersymmetry is broken. It is possible to break supersymmetry spontaneously, but there are reasons [13] for preferring a soft-breaking framework. In such a framework, the terms analogous to (2.3.14) which are added to the MSSM lagrangian are [10]

$$\begin{aligned} \mathcal{L}_{\text{soft}}^{\text{MSSM}} = &-\frac{1}{2}(M_3 \widetilde{G}^a \widetilde{G}^a + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B}\widetilde{B} + \text{h.c.}) \\ &- \left(\widetilde{\overline{u}} A_u \widetilde{Q} \cdot H_u - \widetilde{\overline{d}} A_d \widetilde{Q} \cdot H_d - \widetilde{\overline{e}} A_e \widetilde{L} \cdot H_d\right) \\ &- \widetilde{Q}^* m_Q^2 \widetilde{Q} - \widetilde{L} m_L^2 \widetilde{L} - \widetilde{\overline{u}}^* m_{\overline{u}}^2 \widetilde{\overline{u}} - \widetilde{\overline{e}}^* m_{\overline{e}}^2 \widetilde{\overline{e}} \\ &- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u \cdot H_d + \text{h.c.}) \end{aligned} \tag{2.4.11}$$

where $\widetilde{G}^a$, $\widetilde{W}^a$, and $\widetilde{B}$ are the gauginos of the $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ groups respectively. In the next chapter we will isolate the neutral fields in this term to generate the effective interactions between the symmetry breaking hidden sector and the visible sector of the MSSM. This effective interaction will ultimately allow us to compute the magnetic dipole term which couples photons to goldstinos.

Chapter 3

The Decay in a Toy Model and the MSSM

Before we calculate the rate for a pseudo-goldstino to decay into a gravitino in the MSSM with supergravity, it will be useful to establish some intuition and methodology through studying simpler cases. In this chapter we study a model of two-sector supersmmetry breaking in a toy model with only a vector superfield in the visible sector. We then study the case of multiple supersymmetry breaking in the MSSM without SUGRA. Through these studies we derive expressions for the magnetic dipole coupling between the pseudo-goldstino and the gravitino which will be used to study the principal decay scenario in the next chapter.

3.1 Simple Model of Pseudo-Goldstino to Gravitino Decay

In this section, we present the main problem of thesis in the context of a toy model. Unlike the real problem, the toy model has global instead of local supersymmetry breaking and only has an abelian vector superfield in the visible sector as opposed to the full MSSM. To simplify the particle spectrum, we assume that the visible sector breaks supersymmetry independently of the hidden sector, and hence the vector superfield breaks supersymmetry directly through a Fayet-Iliopolous (FI) term, instead of indirectly through scalar VEVs. We note that this independent breaking in the visible sector is a feature of the toy model which is not present in the MSSM.

We will calculate the magnetic dipole transition rate from a pseudo-goldstino to a goldstino. We study this particular decay because when we consider the real case with supergravity, the goldstino could be interpreted as the longitudinal component of the gravitino and hence this decay models the pseudo-goldstino to gravitino decay of the desired problem. The decay rate in this toy model is defined by an operator of the form

$$\mathcal{L} = -\frac{\Omega_0}{\sqrt{2}}\,(\zeta\,\sigma^{\mu\nu}\eta)F_{\mu\nu} + \text{h.c.}, \tag{3.1.1}$$

where Ω_0 is the inverse mass dimension coupling, ζ is the massive pseudo-goldstino, η is the massless goldstino, and $F_{\mu\nu}$ is the abelian field strength which stands in for electromagnetism in this toy model. From standard computations, we find the associated decay rate is

$$\Gamma(\zeta \to \eta + \gamma) = \frac{m_\zeta^3\,\Omega_0^2}{16\pi}. \tag{3.1.2}$$

Hence, in computing this decay rate we only require two pieces of information: Ω_0 the coefficient of the MDI and m_ζ the mass of the pseudo-goldstino.

We note that when we incorporate supergravity, the goldstino becomes the longitudinal component of the gravitino and assumes a mass of $m_{3/2}$, and the mass of the pseudo-goldstino becomes approximately $2m_{3/2}$[8], and therefore, since the masses of all the particles are fixed in the SUGRA case, the problem of computing the decay rate simply reduces to the problem of finding Ω_0.

3.1.1 The Model

Our goal is to compute the decay rate for a pseudo-goldstino to go to a goldstino via a magnetic dipole interaction in a simple system of two-sector supersymmetry breaking. The simplest such system consists of two chiral superfields $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ which represent the two hidden sectors and a vector superfield $\boldsymbol{V}$ which represents the visible sector. The breaking scenario for this model is represented in Fig. 3-1. The lagrangian for this system is

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \left[\int d^2\theta(\gamma_1\boldsymbol{X}_1 + \gamma_2\boldsymbol{X}_2) + \text{h.c.}\right] + \kappa\int d^4\theta\,\boldsymbol{V} + \mathcal{L}_{\text{int}}, \tag{3.1.3}$$

where $\mathcal{L}_{\text{kin}}$ is the collection of standard kinetic terms for our chiral and vector field multiplets, and the second and third terms ensure the independent breaking of supersymmetry

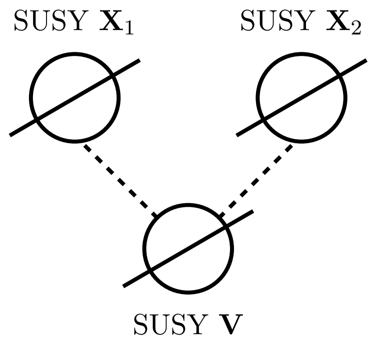

Figure 3-1: Breaking Scenario of Toy Model: We assume that the two hidden sector chiral superfields $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ break SUSY independently of the visible sector vector superfield $\boldsymbol{V}$. An FI term breaks SUSY in the visible sector. Each sector has a single fermion all of which mix to yield the mass eigenstate neutralino, pseudo-goldstino, and goldstino.

in each of sectors. The final term defines the interaction between the sectors. We find this interaction by recognizing that $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are hidden sector fields and hence must interact with the visible sector vector field $\boldsymbol{V}$ through an effective soft-supersymmetry breaking interaction. Namely, they must have an interaction of the form

$$\mathcal{L}_{\text{int}} = -\int d^2\theta \left(\frac{\alpha}{2F_1} \boldsymbol{X}_1 + \frac{\beta}{2F_2} \boldsymbol{X}_2 \right) \boldsymbol{W}^\alpha \boldsymbol{W}_\alpha + \text{h.c}, \tag{3.1.4}$$

where α and β are constants of mass dimension 1, and F_1 and F_2 are the F-component VEVs of $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ respectively. $\mathbf{W}^\alpha$ is the superfield strength of the vector superfield $\boldsymbol{V}$. Since $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are hidden sector fields we will expand them in their nonlinear parameterizations. Namely we have

$$\mathbf{X}_k = \frac{\eta_k \eta_k}{2F_k} + \sqrt{2}\eta_k \theta + \theta^2 F_k, \tag{3.1.5}$$

where $k = 1, 2$.

The first step towards computing the desired decay rate, is to find the interaction lagrangian for the decay process and the mass eigenstate goldstino and pseudo-goldstino written in terms of the fermions of each sector. We can make progress towards both of these goals by expanding the interaction lagrangian (3.1.4) in terms of the component fields. Doing so, employing spinor identities [7], and integrating over spinor space, we find

$$\mathcal{L}_{\text{int}} = -\frac{\alpha}{2}\lambda\lambda + \frac{\alpha D}{\sqrt{2}F_1}\eta_1\lambda - \frac{\alpha}{\sqrt{2}F_1}(\lambda\sigma^{\mu\nu}\eta_1)F_{\mu\nu} - \frac{\alpha D^2}{4F_1^2}\eta_1\eta_1 + (1 \to 2, \alpha \to \beta) + \text{h.c.} + \cdots , \tag{3.1.6}$$

where λ is the gaugino of the vector superfield, and η_1 and η_2 are the fermions for sectors 1 and 2, respectively. Also, we applied a Weyl rotation $\lambda \to i\lambda$ in order to remove the imaginary factors from the interaction coefficients. From the above result we see that the gaugino has a zeroth-order mass $m_\lambda^{(0)} = \alpha + \beta$. The third term above and the corresponding $(1 \to 2, \alpha \to \beta)$ contribution define the magnetic dipole operator which makes our decay possible. Our goal will be to replace the gaugino and the fermion fields of sectors 1 and 2 in this operator with the pseudo-goldstino and goldstino fields. To achieve this, we must express our gaugino and sector fermion fields in terms of the pseudo-goldstino and goldstino fields. Finding such a change of basis formula is grounded in diagonalizing the mass matrix for our system. This mass matrix is found from considering the fermion bilinear terms in the (3.1.6). Specifically we have

$$M_F = \begin{pmatrix} \alpha + \beta & -\frac{\alpha D}{\sqrt{2}F_1} & -\frac{\beta D}{\sqrt{2}F_2} \\ -\frac{\alpha D}{\sqrt{2}F_1} & \frac{\alpha D^2}{2F_1^2} & 0 \\ -\frac{\beta D}{\sqrt{2}F_2} & 0 & \frac{\beta D^2}{2F_2^2} \end{pmatrix}, \tag{3.1.7}$$

where the matrix is written in the λ, η_1, η_2 basis.

3.1.2 Change of Basis Matrix

Our goal is to find the mass eiegnstates of M_F so that we may rewrite the MDI terms in (3.1.6) in terms of the goldstino and pseudo-goldstino states. To this end we need to find the change of basis matrix from the $(\lambda, \eta_1, \eta_2)$ basis to (χ, ζ, η) where χ is a mass eigenstate neutralino, ζ is the pseudo-goldstino, and η is the actual goldstino. From the theory of supersymmetry breaking we already know the vector representation of the goldstino (2.3.7) in this system

$$|\eta\rangle = \frac{1}{F_\eta}\begin{pmatrix} D/\sqrt{2} \\ F_1 \\ F_2 \end{pmatrix}. \tag{3.1.8}$$

Applying the neutralino mass matrix to this vector confirms that the vector defines a massless state. However, an attempt to compute the other mass eiegnstates reveals that they do not have such simple forms. Indeed this toy system is exactly soluble, but the linear combinations representing the mass eigenstates are conceptually opaque. Consequently, instead of solving for these eiegnstates immediately, we treat the coefficients which define the change of basis formula as "black boxes" which will be filled in later. With this perspective we can easily write down the change of basis equation. We have

$$\begin{pmatrix} \lambda \\ \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \Theta_{\lambda\chi} & \Theta_{\lambda\zeta} & \Theta_{\lambda\eta} \\ \Theta_{1\chi} & \Theta_{1\zeta} & \Theta_{1\eta} \\ \Theta_{2\chi} & \Theta_{2\zeta} & \Theta_{2\eta} \end{pmatrix} \begin{pmatrix} \chi \\ \zeta \\ \eta \end{pmatrix}, \tag{3.1.9}$$

where the 3×3 matrix is orthogonal.

3.1.3 MDI and Decay Rate

With the above change of basis matrix we can write down an open-form expression for the magnetic dipole interaction between the goldstino and pseudo-goldstino. Using the third term in (3.1.6) we have

$$\mathcal{L} = -\frac{1}{\sqrt{2}} \left[\frac{\alpha}{F_1} (\lambda \sigma^{\mu\nu} \eta_1) F_{\mu\nu} + \frac{\beta}{F_2} (\lambda \sigma^{\mu\nu} \eta_2) F_{\mu\nu} \right] + \cdots \tag{3.1.10}$$

$$\equiv -\frac{\Omega_0}{\sqrt{2}} (\zeta \sigma^{\mu\nu} \eta) F_{\mu\nu} + \cdots , \tag{3.1.11}$$

where

$$\Omega_0 = \frac{\alpha}{F_1}(\Theta_{\lambda\zeta}\Theta_{1\eta} - \Theta_{\lambda\eta}\Theta_{1\zeta}) + \frac{\beta}{F_2}(\Theta_{\lambda\zeta}\Theta_{2\eta} - \Theta_{\lambda\eta}\Theta_{2\zeta}). \tag{3.1.12}$$

We note that the orthogonality of the λ, η_1, and η_2 states forbids the possible "self-MDI" arising from the above change of basis. We now have the magnetic dipole coupling Ω_0 written in terms of the mixing angles between the individual particle states, and according to (3.1.2) to find the decay rate for $\zeta \to \eta + \gamma$ we need only compute these mixing angles and compute the mass of the pseudo-goldstino m_ζ. Using the mass matrix (3.1.7), it is possible to compute m_ζ and Ω_0 exactly, but in preparation for the real case of the MSSM, we will compute each quantity as an expansion. When we consider the MSSM, we will take the SUSY breaking scale in the visible sector to be much less than the SUSY breaking scale in the hidden sector. In this toy model, this scaling corresponds to the limit where $D \ll F_1, F_2$. Taking this limit we can use *Mathematica* to compute the mass of the pseudo goldstino and the coefficient of Ω_0 as an expansion in D. In this calculation we define the pseudo-goldstino as the state which reduces to the eiegenvector $(0, F_2, -F_1)/\sqrt{F_1^2+F_2^2}$ in the limit as $D \to 0$. Then for the pseudo-goldstino mass we have

$$m_\zeta = \frac{\alpha\beta}{2(\alpha+\beta)} \frac{F_{\text{Eff}}^2 D^2}{F_1^2 F_2^2} + O(D^4), \tag{3.1.13}$$

where $F_{\text{Eff}} = \sqrt{F_1^2+F_2^2}$, and for the MDI coefficient we find

$$\Omega_0 = \frac{\alpha\beta}{2\sqrt{2}(\alpha+\beta)^2} \left(\alpha\frac{F_2}{F_1} - \beta\frac{F_1}{F_2}\right) \frac{D^3}{F_1^2 F_2^2} + O(D^5). \tag{3.1.14}$$

Now, using (3.1.2) we can compute the decay rate for $\zeta \to \eta + \gamma$ as a function of the mass scales α and β and the supersymmetry breaking constants F_1 and F_2.

With our final results computed, we can now compare them to our expectations of limiting cases. If we take $D \to 0$, we find that the mass of the pseudo-goldstino goes to zero. This result is to be expected because D provides the mass couplings between the individual fermions from each sector. Hence, if $D \to 0$ the fermions in the hidden sector become sequestered from the gaugino in the visible sector, and we simply have a massive gaugino and two massless goldstinos.

In the limit in which one hidden sector decouples from the visible sector (e.g. $\alpha \to 0$), we find that the mass of the pseudo-goldstino goes to zero. This is also to be expected because if one hidden sector decouples from the visible sector, we simply have two independent sectors of SUSY breaking and hence two goldstinos plus a massive neutralino. One goldstino is associated with the independent hidden sector, and the other goldstino is associated with the remaining hidden sector-visible sector system. This situation is depicted in Fig. 3-2

We will later find that the coefficient Ω_0 is proportional to the pseudo-goldstino mass m_ζ, so that many of the arguments justifying the limiting cases of Ω_0 simply echo the arguments for the limiting cases of m_ζ. However, we must note that when we incorporate supergravity, the mass of the pseudo-goldstino will then be independent of the couplings between the sectors and hence the coefficient Ω_0 will remain zero for limiting cases similar to those cited above. One interesting distinction between Ω_0 and m_ζ in this case is that

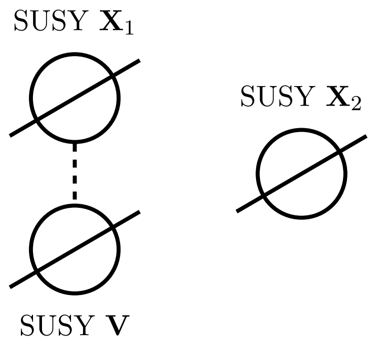

Figure 3-2: Decoupling of One Sector: When we decouple one sector from the visible sector, we simply have two distinct and unmixed sectors of supersymmetry breaking. The result is that sector 2, obtains a massless goldstino, and sector 1 together with the visible sector obtains a massless goldstino and a massive neutralino. Hence there is no state corresponding to the pseudo-goldstino.

(3.1.14) shows that Ω_0, unlike m_ζ, goes to zero as we take $\alpha \to \beta$ and $F_1 \to F_2$.

3.1.4 Degenerate Perturbation Theory

Although *Mathematica* is able to easily solve for the eigenspectrum of this toy model based on a 3×3 matrix, when we move to the MSSM and have to consider a 6×6 matrix the algebraic reduction of the eigenspectrum is more difficult. So it would prove useful to obtain the pseudo-goldtsino mass (3.1.13) and the magnetic dipole interaction coefficient (3.1.14) through an analytic method. In this section we fulfill the first of these goals by computing the mass of the pseudo-goldstino using degenerate perturbation theory.

We begin with the mass matrix for this system

$$M_F = \begin{pmatrix} \alpha+\beta & -\frac{\alpha D}{\sqrt{2}F_1} & -\frac{\beta D}{\sqrt{2}F_2} \\ -\frac{\alpha D}{\sqrt{2}F_1} & \frac{\alpha D^2}{2F_1^2} & 0 \\ -\frac{\beta D}{\sqrt{2}F_2} & 0 & \frac{\beta D^2}{2F_2^2} \end{pmatrix}. \tag{3.1.15}$$

For perturbation theory, we must select out the zeroth order matrix. In the limit $D \ll F_1, F_2$

the choice is obvious. We have

$$M^{(0)} = \begin{pmatrix} \alpha+\beta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{3.1.16}$$

and the first and second order matrices are, respectively,

$$M^{(1)} = \begin{pmatrix} 0 & -\frac{\alpha D}{\sqrt{2}F_1} & -\frac{\beta D}{\sqrt{2}F_2} \\ -\frac{\alpha D}{\sqrt{2}F_1} & 0 & 0 \\ -\frac{\beta D}{\sqrt{2}F_2} & 0 & 0 \end{pmatrix}, \qquad M^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\alpha D^2}{2F_1^2} & 0 \\ 0 & 0 & \frac{\beta D^2}{2F_2^2} \end{pmatrix}. \tag{3.1.17}$$

From (3.1.16) the zeroth order spectrum and eigenvectors are easily found to be

$$m_\lambda^{(0)} = \alpha + \beta, \tag{3.1.18}$$

$$m_1^{(0)} = 0, \tag{3.1.19}$$

$$m_2^{(0)} = 0, \tag{3.1.20}$$

where the eiegenstates $|\lambda\rangle$ $|\eta_1\rangle$ and $|\eta_2\rangle$ are just the standard basis vectors for a 3×3 matrix. From the zeroth order spectrum, it is clear that degenerate perturbation theory will be necessary to compute the higher order terms. From [15] the general formula for a Hamiltonian matrix element computed using second-order degenerate perturbation theory is

$$\begin{aligned} \Delta_{kl} &= \Delta_{kl}^{(1)} + \Delta_{kl}^{(2)} + \cdots \\ &= \langle k|V|l\rangle + \sum_{m\neq D} \frac{\langle k|V|m\rangle\langle m|V|l\rangle}{E_D - E_m} + \cdots, \end{aligned} \tag{3.1.21}$$

where V is the perturbation, $|k\rangle$ are $|l\rangle$ are eiegenstates in the degenerate state space D, and E_D is the common energy of that state space. The notation $k \neq D$ means that the sum runs over all the states *outside* the degenerate state space. A quick application of this formula to (3.1.17) shows that there is no first-order correction to the spectrum, but using the first order matrix $M^{(1)}$ in the second term of (3.1.21) and the second order matrix $M^{(2)}$

in the first term, we obtain a new perturbation matrix for the degenerate spectrum:

$$\Delta^{(2)}_{\text{EFF}} = \begin{pmatrix} \dfrac{\alpha D^2}{2F_1^2} - \dfrac{\alpha^2 D^2}{2F_1^2(\alpha+\beta)} & -\dfrac{\alpha\beta D^2}{2F_1F_2(\alpha+\beta)} \\ -\dfrac{\alpha\beta D^2}{2F_1F_2(\alpha+\beta)} & \dfrac{\beta D^2}{2F_2^2} - \dfrac{\beta^2 D^2}{2F_2^2(\alpha+\beta)} \end{pmatrix} \tag{3.1.22}$$

$$= \frac{\alpha\beta D^2}{2F_1^2F_2^2(\alpha+\beta)} \begin{pmatrix} F_2^2 & -F_1F_2 \\ -F_1F_2 & F_1^2 \end{pmatrix}. \tag{3.1.23}$$

Solving for the eiegenstates and the corresponding eiegenvalues we find

$$|\eta\rangle = \frac{1}{F_{\text{Eff}}} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \qquad m_\eta = 0, \tag{3.1.24}$$

$$|\zeta\rangle = \frac{1}{F_{\text{Eff}}} \begin{pmatrix} F_2 \\ -F_1 \end{pmatrix}, \qquad m_\zeta = \frac{\alpha\beta}{2(\alpha+\beta)} \left(\frac{F_1^2+F_2^2}{F_1^2F_2^2} \right) D^2. \tag{3.1.25}$$

As expected the goldstino state $|\eta\rangle$ is massless. The state $|\zeta\rangle$ is our pseudo-goldstino state and it has the mass cited in (3.1.13).

Next, we would like to analytically calculate the coefficient Ω_0, but the *Mathematica* result (3.1.14) reveals that it would require third order perturbation theory to compute to lowest order. This is a calculation we could complete in this toy model but would not want to extrapolate to the case of the MSSM, so in the next section we derive a formula which reduces the order of perturbation theory necessary to obtain Ω_0.

3.1.5 Magnetic Dipole Interaction: Supercurrent Derivation

We previously obtained the magnetic dipole interaction between the pseudo-goldstino and the goldstino in a toy model of multiple SUSY breaking. To compute the decay rate for this interaction one must know the mass of the pseudo-goldstino and the value of the magnetic dipole coupling. It is possible to compute both quantities using degenerate perturbation theory, but *Mathematica* reveals that the magnetic dipole coupling is of third order in D and hence would require third order degenerate perturbation theory to compute analytically.

In this section we derive a formula which simplifies the calculation of the magnetic dipole coupling. To derive (3.1.12) we began with the effective lagrangian for soft supersymmetry breaking (3.1.4). However, the goldstino is a special field in the context of supersymmetry breaking and its interaction with the other component fields is constrained by the conservation of the supercurrent. Therefore the more natural method for finding the coupling between the goldstino and the pseudo-goldstino is to simply use the goldstino equation of motion. We proceed accordingly.

We begin with the most general conserved supercurrent for a supersymmetric gauge theory [7]:

$$\begin{aligned} J^\mu_\alpha &= \frac{i}{\sqrt{2}} D^a (\sigma^\mu \lambda^{\dagger\, a})_\alpha + F_k\, i(\sigma^\mu \psi^{\dagger\, k})_\alpha \\ &\quad + (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha D_\nu \phi^{*\, i} - \frac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger\, a})_\alpha F^a_{\nu\rho}, \end{aligned} \tag{3.1.26}$$

where λ^a is a gaugino from the (non-abelian) vector multiplet and ψ_k is a fermion from a chiral multiplet k. From the definition of the goldstino, we can rewrite this result as

$$\begin{aligned} J^\mu_\alpha &= iF_\eta\, (\sigma^\mu \overline{\eta})_\alpha + (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha D_\nu \phi^{*\, i} - \frac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger\, a})_\alpha F^a_{\nu\rho} \\ &\equiv iF_\eta\, (\sigma^\mu \overline{\eta})_\alpha + j^\mu_\alpha, \end{aligned} \tag{3.1.27}$$

where $F_\eta^2 = \sum_k |F_k|^2 + \sum_a D^a D^a/2$, and we defined

$$j^\mu_\alpha \equiv (\sigma^\nu \overline{\sigma}^\mu \psi_i)_\alpha D_\nu \phi^{*\, i} - \frac{1}{2\sqrt{2}} (\sigma^\nu \overline{\sigma}^\rho \sigma^\mu \lambda^{\dagger\, a})_\alpha F^a_{\nu\rho}. \tag{3.1.28}$$

The conservation of J^μ_α then yields

$$\partial_\mu J^\mu_\alpha = iF_\eta (\sigma^\mu \partial_\mu \overline{\eta}) + \partial_\mu j^\mu_\alpha = 0. \tag{3.1.29}$$

In addition to a conservation equation, we can interpret this result as an equation of motion for the goldstino. We can then reverse construct the lagrangian from which this equation could have been derived. We find

$$\mathcal{L}_\eta = i\eta\, \sigma^\mu \partial_\mu \overline{\eta} + \frac{1}{F_\eta} (\eta\, \partial_\mu j^\mu + \text{h.c.}). \tag{3.1.30}$$

The first term is the standard fermion kinetic term and the second term is the goldstino interaction term. This interaction term defines all interactions between the goldstino and the partner-superpartner fields in a supersymmetric theory, regardless of how the goldstino field was originally introduced. For the toy model considered in the previous section, the only partner-superpartner fields occur in the vector multiplet in the visible sector. Hence for the goldstino interaction lagrangian we have

$$\mathcal{L}_{\rm int} = \frac{1}{F_\eta}\,\eta\,\partial_\mu j^\mu + \text{h.c.} \tag{3.1.31}$$

$$= -\frac{1}{2\sqrt{2}F_\eta}\,\eta\,\sigma^\nu\overline{\sigma}^\rho\sigma^\mu(\partial_\mu\overline{\lambda})\,F_{\nu\rho} + \text{h.c.} \equiv \mathcal{L}_{\rm MDI}, \tag{3.1.32}$$

where we used the identity

$$\sigma^\nu\overline{\sigma}^\rho\sigma^\mu = -\eta^{\nu\mu}\sigma^\rho + \eta^{\rho\mu}\sigma^\nu + \eta^{\nu\rho}\sigma^\mu + i\,\epsilon^{\nu\rho\mu\kappa}\sigma_\kappa, \tag{3.1.33}$$

the source free equations of motion of A^μ, and the Bianchi identity to eliminate the second term in Eq. **??** resulting from the product rule. Now using $\sigma^{\nu\rho} = \frac{i}{4}(\sigma^\nu\overline{\sigma}^\rho - \sigma^\rho\overline{\sigma}^\nu)$ and the antisymmetrization property of $F_{\nu\rho}$ we have

$$\mathcal{L}_{\rm MDI} = \frac{1}{\sqrt{2}F_\eta}\eta\,\sigma^{\nu\rho}\sigma^\mu\partial_\mu\overline{\lambda}F_{\nu\rho} + \text{h.c.}, \tag{3.1.34}$$

where we we Weyl rotated $\overline{\lambda} \to -i\overline{\lambda}$ to eliminate the imaginary factor. Due to the interaction between the two hidden sectors and the visible sector, λ is not a mass eigenstate and therefore does not satisfy an independent equation of motion. We can, however, decompose λ into a linear combination of mass eigenstates one of which is the pseudo-goldstino. Doing so we have

$$\lambda = \Theta_{\lambda\chi}\,\chi + \Theta_{\lambda\eta}\,\eta + \Theta_{\lambda\zeta}\,\zeta, \tag{3.1.35}$$

where χ, η, ζ are the neutralino, goldstino, and pseudo-goldstino respectively. Substituting (3.1.35) into (3.1.34) and focusing on the pseudo-goldstino contribution to λ we obtain

$$\mathcal{L}_{\eta\,\rm MDI} = \frac{\Theta_{\lambda\zeta}}{\sqrt{2}F_\eta}\,\eta\,\sigma^{\nu\rho}\sigma^\mu\partial_\mu\overline{\zeta}\,F_{\nu\rho} + \text{h.c.}. \tag{3.1.36}$$

Using the equation of motion of ζ we find, finally,

$$\mathcal{L}_{\eta\,\rm MDI} = -\frac{m_\zeta\,\Theta_{\lambda\zeta}}{F_\eta,}\,\overline{\zeta}\,\sigma^{\nu\rho}\eta\,F_{\nu\rho} + \text{h.c.}, \tag{3.1.37}$$

the magnetic dipole interaction between the pseudo-goldstino and the goldstino. Conservation of the supercurrent requires the goldstino interactions to be the same no matter how they are derived. Therefore we can equate (3.1.37) to (3.1.1). This gives us the identity

$$\frac{m_\zeta\,\Theta_{\lambda\,\zeta}}{\sqrt{2}F_\eta} = \frac{\alpha}{F_1}(\Theta_{\lambda\,\zeta}\Theta_{1\,\eta} - \Theta_{\lambda\,\eta}\Theta_{1\,\zeta}) + \frac{\beta}{F_2}(\Theta_{\lambda\,\zeta}\Theta_{2\,\eta} - \Theta_{\lambda\,\eta}\Theta_{2\,\zeta}). \tag{3.1.38}$$

or the simply the magnetic coupling definition

$$\Omega_0 = \frac{m_\zeta\,\Theta_{\lambda\,\zeta}}{F_\eta}. \tag{3.1.39}$$

The left side of (3.1.38) is much easier to calculate analytically than the right side because after we obtain m_ζ (which is of order D^2) we need only calculate $\Theta_{\lambda\,\zeta}$ to first order in D in order to reproduce *Mathematica*'s result that Ω_0 is of order D^3. The calculation is straight forward so we provide it here. Using $|\lambda\rangle = (1,0,0)$ and $|\zeta^{(0)}\rangle = (0, F_2, -F_1)/F_{\text{Eff}}$ we have

$$\begin{aligned}
\Theta_{\lambda\,\zeta} &= \langle\lambda|\zeta\rangle \\
&= \langle\lambda|\zeta^{(0)}\rangle + \frac{\langle\lambda|M^{(1)}|\zeta^0\rangle}{m_\zeta^{(0)} - m_\lambda^{(0)}} + \cdots \\
&= \frac{1}{\sqrt{2}(\alpha+\beta)}\left(\alpha\frac{F_2}{F_1} - \beta\frac{F_1}{F_2}\right)\frac{D}{F_{\text{Eff}}} + \cdots\,.
\end{aligned} \tag{3.1.40}$$

Using the pseudo-goldstino mass result (3.1.13) and the expansion $\dfrac{1}{F_\eta} = \dfrac{1}{F_{\text{Eff}}} + O(D^2)$ we then find for Ω_0

$$\Omega_0 = \frac{\alpha\beta}{2\sqrt{2}(\alpha+\beta)^2}\left(\alpha\frac{F_2}{F_1} - \beta\frac{F_1}{F_2}\right)\frac{D^3}{F_1^2F_2^2} + O(D^5), \tag{3.1.41}$$

in direct agreement with (3.1.14). This agreement shows the correctness of this analysis. When we consider the MSSM, our neutralino mass matrix will be too complex to analyze using the series expansion in *Mathematica*, so we will use this method to obtain the interaction coefficients.

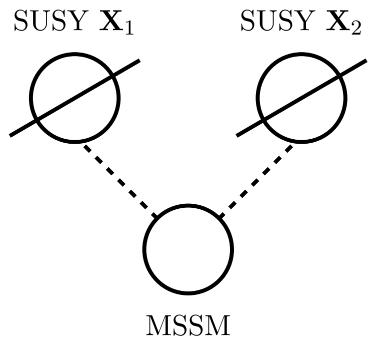

Figure 3-3: Breaking Scenario of MSSM: Unlike in the toy model, the visible sector of the MSSM does not break supersymmetry independent of the couplings to the hidden sectors.

3.2 Pseudo-Goldstino to Gravitino Decay in the MSSM

In this section we construct the neutralino mass matrix with two hidden sectors in the MSSM and we will use the result to compute the coupling of the magnetic dipole interaction between the pseudo-goldstino and the goldstino. We first begin by constructing the 5×5 matrix which results from the inclusion of a single hidden sector, and we then use the result to extrapolate to the case of two hidden sectors. The breaking scenario is depicted in Fig. 3-3. In parallel to our study of the toy model, we will compute the coupling of the magnetic dipole interaction as a function of the mixing angles and parameters of the MSSM.

3.2.1 Neutralino Mass Matrix for One Hidden Sector

We begin with the $\mathcal{L}_{\text{soft}}$ for the MSSM, reproduced here for convenience:

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} = &-\frac{1}{2}(M_3\widetilde{G}^a\widetilde{G}^a + M_2\widetilde{W}^a\widetilde{W}^a + M_1\widetilde{B}\widetilde{B} + \text{h.c.})\\ &-\left(\widetilde{\overline{u}}A_u\widetilde{Q}\cdot H_u - \widetilde{\overline{d}}A_d\widetilde{Q}\cdot H_d - \widetilde{\overline{e}}A_e\widetilde{L}\cdot H_d\right)\\ &-\widetilde{Q}^* m_Q^2\widetilde{Q} - \widetilde{L}m_L^2\widetilde{L} - \widetilde{\overline{u}}^* m_{\overline{u}}^2\widetilde{\overline{u}} - \widetilde{\overline{e}}^* m_{\overline{e}}^2\widetilde{\overline{e}}\\ &-m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_u\cdot H_d + \text{h.c.}) \qquad (3.2.1)\end{aligned}$$

Toward finding the magnetic dipole interaction, we want to isolate the terms which contain neutral particles. Doing so we have

$$\begin{aligned}\mathcal{L}_{\text{soft}}^{\text{MSSM}} &= -m_{H_u}^2 H_u^{0\,\dagger} H_u^0 - m_{H_d}^2 H_d^{0\,\dagger} H_d^0 - \frac{1}{2}(M_2 \widetilde{W}^3 \widetilde{W}^3 + M_1 \widetilde{B}\widetilde{B} + \text{h.c}) \\ &\quad + (b H_u^0 H_d^0 + \text{h.c.}) + \cdots . \end{aligned} \tag{3.2.2}$$

Next, we include the effects of a hidden sector by taking the above soft terms to arise from interactions with a hidden sector. The general way to write down such interactions was outlined in Section 2.3.3 and by direct analogy we can write the corresponding interaction for (3.2.2) as

$$\begin{aligned}\mathcal{L}_{\text{soft 1 sector}}^{\text{MSSM}} &= -\int d^4\theta \, \frac{\mathbf{X}^\dagger \mathbf{X}}{F_X^2} \left[m_{H_u}^2 \mathbf{H}_u^{0\,\dagger} \mathbf{H}_u^0 + m_{H_d}^2 \mathbf{H}_d^{0\,\dagger} \mathbf{H}_d^0 \right] \\ &\quad - \left(\int d^2\theta \, \frac{\mathbf{X}}{F_X} \left[\frac{M_2}{2} \mathbf{W}^{\alpha 3} \mathbf{W}_\alpha^3 + \frac{M_1}{2} \mathbf{B}^\alpha \mathbf{B}_\alpha - b \, \mathbf{H}_u^0 \mathbf{H}_d^0 \right] + \text{h.c.} \right), \end{aligned} \tag{3.2.3}$$

where the boldface type once again denotes superfields. Now, we integrate the above lagrangian and focus on the fermion bilinears to obtain

$$\begin{aligned}\mathcal{L}_{\text{soft 1 sector}}^{\text{MSSM}} &= \frac{v_u}{\sqrt{2}} \left(\frac{m_{H_u}^2}{F_X} \right) \eta \widetilde{H}_u^0 - \frac{v_u}{2\sqrt{2}} \left(\frac{m_{H_u}^2 F_u}{F_X^2} \right) \eta\eta + \frac{v_d}{\sqrt{2}} \left(\frac{m_{H_d}^2}{F_X} \right) \eta \widetilde{H}_d^0 - \frac{v_d}{2\sqrt{2}} \left(\frac{m_{H_d}^2 F_d}{F_X^2} \right) \eta\eta \\ &\quad + \frac{M_2}{\sqrt{2}} \left(\frac{D^3}{F_X} \right) \eta \widetilde{W}^3 - \frac{1}{2} M_2 \widetilde{W}^3 \widetilde{W}^3 - \frac{1}{4} M_2 \left(\frac{D^3}{F_X} \right)^2 \eta\eta \\ &\quad + \frac{M_1}{\sqrt{2}} \left(\frac{D^Y}{F_X} \right) \eta \widetilde{B} - \frac{1}{2} M_1 \widetilde{B}\widetilde{B} - \frac{1}{4} M_1 \left(\frac{D^Y}{F_X} \right)^2 \eta\eta \\ &\quad - \frac{v_d}{\sqrt{2}} \left(\frac{b}{F_X} \right) \eta \widetilde{H}_u^0 - \frac{v_u}{\sqrt{2}} \left(\frac{b}{F_X} \right) \eta \widetilde{H}_d^0 + \frac{b}{2\sqrt{2} F_X} \left(v_u \frac{F_d}{F_X} + v_d \frac{F_u}{F_X} \right) \eta\eta. \end{aligned} \tag{3.2.4}$$

And from this result we can write down the 5×5 neutralino mass matrix which includes the mass interactions with the hidden sector fermion η:

$$M_{\tilde{N}^0}^{5\times 5} = \begin{pmatrix} M_1 & 0 & g_Y v_u/\sqrt{2} & -g_Y v_d/\sqrt{2} & (M_{\tilde{N}^0})_{51} \\ 0 & M_2 & -g v_u/\sqrt{2} & g v_d/\sqrt{2} & (M_{\tilde{N}^0})_{52} \\ g_Y v_u/\sqrt{2} & -g v_u/\sqrt{2} & 0 & -\mu & (M_{\tilde{N}^0})_{53} \\ -g_Y v_d/\sqrt{2} & g v_d/\sqrt{2} & -\mu & 0 & (M_{\tilde{N}^0})_{54} \\ (M_{\tilde{N}^0})^*_{51} & (M_{\tilde{N}^0})^*_{52} & (M_{\tilde{N}^0})^*_{53} & (M_{\tilde{N}^0})^*_{54} & (M_{\tilde{N}^0})_{55} \end{pmatrix}, \tag{3.2.5}$$

where

$$\begin{pmatrix} (M_{\tilde{N}^0})_{51} \\ (M_{\tilde{N}^0})_{52} \\ (M_{\tilde{N}^0})_{53} \\ (M_{\tilde{N}^0})_{54} \end{pmatrix} = \begin{pmatrix} -\frac{M_1}{\sqrt{2}}\frac{D^Y}{F_X} \\ -\frac{M_2}{\sqrt{2}}\frac{D^3}{F_X} \\ -\frac{1}{\sqrt{2}F_X}\left(v_u m_{H_u}^2 - v_d b\right) \\ -\frac{1}{\sqrt{2}F_X}\left(v_d m_{H_d}^2 - v_u b\right) \end{pmatrix}, \tag{3.2.6}$$

and

$$(M_{\tilde{N}^0})_{55} = \frac{M_1}{2}\left(\frac{D^Y}{F_X}\right)^2 + \frac{M_2}{2}\left(\frac{D^3}{F_X}\right)^2 + \frac{F_d}{\sqrt{2}F_X^2}(v_d m_{H_d}^2 - v_u b) + \frac{F_u}{\sqrt{2}F_X^2}(v_u m_{H_u}^2 - v_d b). \tag{3.2.7}$$

3.2.2 Neutralino Mass Matrix for Two Hidden Sectors

It is easy to extrapolate the above results to the case of two sectors. We begin by modifying the soft lagrangian in (3.2.3) to include two hidden sector fields. To prevent confusion with the soft mass terms of the wino and bino, we label these two sectors by A and B rather than 1 and 2. Our effective lagrangian is then

$$\begin{aligned} \mathcal{L}^{\text{MSSM}}_{\text{soft 2 sectors}} = &- \int d^4\theta\, \frac{\mathbf{X}_A^\dagger \mathbf{X}_A}{F_A^2}\left[\alpha_{H_u}^2 \mathbf{H}_u^{0\,\dagger}\mathbf{H}_u^0 + \alpha_{H_d}^2 \mathbf{H}_d^{0\,\dagger}\mathbf{H}_d^0\right] \\ &- \left(\int d^2\theta\, \frac{\mathbf{X}_A}{F_A}\left[\frac{M_{A2}}{2}\mathbf{W}^{\alpha 3}\mathbf{W}^3_\alpha + \frac{M_{A1}}{2}\mathbf{B}^\alpha \mathbf{B}_\alpha - b_A\, \mathbf{H}_u^0 \mathbf{H}_d^0\right] + \text{h.c.}\right) \\ &+ (A \to B, \alpha \to \beta). \end{aligned} \tag{3.2.8}$$

We can then write the mass coefficients in (3.2.2), in terms of the parameters of this lagrangian. Taking the hidden sector fields to be at their VEVs we have

$$M_1 = M_{A1} + M_{B1}, \tag{3.2.9}$$

$$M_2 = M_{A2} + M_{B2}, \tag{3.2.10}$$

$$m^2_{H_d} = \alpha^2_{H_d} + \beta^2_{H_d}, \tag{3.2.11}$$

$$m^2_{H_u} = \alpha^2_{H_u} + \beta^2_{H_u}, \tag{3.2.12}$$

$$b = b_A + b_B. \tag{3.2.13}$$

With (3.2.8) it is then a simple matter to write down the matrix elements for the 6×6 neutralino mass matrix. In direct analogy to (3.2.5) we have

$$M^{6\times 6}_{\widetilde{N}^0} = \begin{pmatrix} M^{4\times 4}_{\widetilde{N}^0} & \Gamma^{4\times 1}_A & \Gamma^{4\times 1}_B \\ (\Gamma^{4\times 1}_A)^\dagger & (M_{\widetilde{N}^0})_{AA} & 0 \\ (\Gamma^{4\times 1}_B)^\dagger & 0 & (M_{\widetilde{N}^0})_{BB} \end{pmatrix}, \tag{3.2.14}$$

where $M^{4\times 4}_{\widetilde{N}^0}$ is the 4×4 submatrix of (3.2.5),

$$\begin{pmatrix} \Gamma^{4\times 1}_A & \Gamma^{4\times 1}_B \end{pmatrix} = \begin{pmatrix} -\frac{M_{A1}}{\sqrt{2}}\frac{D^Y}{F_A} & -\frac{M_{B1}}{\sqrt{2}}\frac{D^Y}{F_B} \\ -\frac{M_{A2}}{\sqrt{2}}\frac{D^3}{F_A} & -\frac{M_{B2}}{\sqrt{2}}\frac{D^3}{F_B} \\ -\frac{1}{\sqrt{2}F_A}\left(v_u\alpha^2_{H_u} - v_d b_A\right) & -\frac{1}{\sqrt{2}F_B}\left(v_u\beta^2_{H_u} - v_d b_B\right) \\ -\frac{1}{\sqrt{2}F_A}\left(v_d\alpha^2_{H_d} - v_u b_A\right) & -\frac{1}{\sqrt{2}F_B}\left(v_d\beta^2_{H_d} - v_u b_B\right) \end{pmatrix}, \tag{3.2.15}$$

and

$$(M_{\widetilde{N}^0})_{AA} = \frac{M_{A1}}{2}\left(\frac{D^Y}{F_A}\right)^2 + \frac{M_{A2}}{2}\left(\frac{D^3}{F_A}\right)^2 + \frac{F_d}{\sqrt{2}F_A^2}(v_d\alpha^2_{H_d} - v_u b_A) + \frac{F_u}{\sqrt{2}F_A^2}(v_u\alpha^2_{H_u} - v_d b_A), \tag{3.2.16}$$

$$(M_{\widetilde{N}^0})_{BB} = \frac{M_{B1}}{2}\left(\frac{D^Y}{F_B}\right)^2 + \frac{M_{B2}}{2}\left(\frac{D^3}{F_B}\right)^2 + \frac{F_d}{\sqrt{2}F_B^2}(v_d\beta^2_{H_d} - v_u b_B) + \frac{F_u}{\sqrt{2}F_B^2}(v_u\beta^2_{H_u} - v_d b_B). \tag{3.2.17}$$

With this mass matrix completely written out we can now turn to a calculation of the magnetic dipole coupling between the pseudo-goldstino and goldstino.

3.2.3 Perturbation Theory

Following our analysis of the toy model, we can use (3.2.8) to compute the rate for a pseudo-goldstino to decay to a goldstino via a magnetic dipole transition in the MSSM. To this end we must determine the coefficient of the magnetic dipole operator and the mass of the pseudo-goldstino.

First, we consider the coefficient of the magnetic dipole interaction. To compute this coefficient we must first write the change of basis equations from the $(\widetilde{W}^3, \widetilde{B}, \widetilde{H}_u^0, \widetilde{H}_d^0, \eta_A, \eta_B)$ basis to the $(\widetilde{N}_1^0, \widetilde{N}_2^0, \widetilde{N}_3^0, \widetilde{N}_4^0, \zeta, \eta)$ basis. Because the magnetic dipole coupling only contains interactions between the neutral gauginos and fermions of the hidden sector, we don't need to consider the change of basis equations for $\widetilde{H}_u^0$ or $\widetilde{H}_d^0$. Moreover, the decay is from a pseudo-goldstino to a goldstino so we only need to focus on the goldstino and pseudo-goldstino contributions. Therefore, the relevant change of basis equations are

$$\widetilde{W}^3 = \Theta_{\widetilde{W}^3\,\eta}\eta + \Theta_{\widetilde{W}^3\,\zeta}\zeta + \cdots\,, \tag{3.2.18}$$

$$\widetilde{B} = \Theta_{\widetilde{B}\,\eta}\eta + \Theta_{\widetilde{W}^3\,\zeta}\zeta + \cdots\,, \tag{3.2.19}$$

$$\eta_A = \Theta_{A\,\eta}\eta + \Theta_{A\,\zeta}\zeta + \cdots\,, \tag{3.2.20}$$

$$\eta_B = \Theta_{B\,\eta}\eta + \Theta_{B\,\zeta}\zeta + \cdots\,, \tag{3.2.21}$$

where $\cdots$ are the mass eigenstate neutralino terms which are irrelevant for this analysis. Now, taking the last two sets of terms in (3.2.8), the change of basis equations (3.2.18)-(3.2.21), and the definition of the photon gauge field in terms of the $\mathrm{SU}(2)_L{\times}\mathrm{U}(1)_Y$ gauge fields we have

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} &= -\Bigg(\int d^2\theta\, \frac{\mathbf{X}_A}{F_A}\left[\frac{M_{A2}}{2}\mathbf{W}^{\alpha 3}\mathbf{W}^3_\alpha + \frac{M_{A1}}{2}\mathbf{B}^\alpha\mathbf{B}_\alpha\right] \\
&\qquad\qquad + \frac{\mathbf{X}_B}{F_B}\left[\frac{M_{B2}}{2}\mathbf{W}^{\alpha 3}\mathbf{W}^3_\alpha + \frac{M_{B1}}{2}\mathbf{B}^\alpha\mathbf{B}_\alpha\right] + \text{h.c.}\Bigg) + \cdots \\
&= -\frac{1}{\sqrt{2}F_A}\left(M_{A2}W^3_{\mu\nu}\widetilde{W}^3 + M_{A1}B_{\mu\nu}\widetilde{B}\right)\sigma^{\mu\nu}\eta_A \\
&\quad - \frac{1}{\sqrt{2}F_B}\left(M_{B2}W^3_{\mu\nu}\widetilde{W}^3 + M_{B1}B_{\mu\nu}\widetilde{B}\right)\sigma^{\mu\nu}\eta_B + \cdots \\
&\equiv -\frac{\Omega_1}{\sqrt{2}}(\zeta\,\sigma^{\mu\nu}\eta)F_{\mu\nu} + \cdots
\end{aligned}
\tag{3.2.22}
$$

where

$$
\begin{aligned}
\Omega_1 = \frac{1}{F_A}&\left[M_{A2}\sin\theta_W\left(\Theta_{\widetilde{W}^3\,\zeta}\Theta_{A\,\eta} - \Theta_{\widetilde{W}^3\,\eta}\Theta_{A\,\zeta}\right) + M_{A1}\cos\theta_W\left(\Theta_{\widetilde{B}\,\zeta}\Theta_{A\,\eta} - \Theta_{\widetilde{B}\,\eta}\Theta_{A\,\zeta}\right)\right] \\
+ \frac{1}{F_B}&\left[M_{B2}\sin\theta_W\left(\Theta_{\widetilde{W}^3\,\zeta}\Theta_{B\,\eta} - \Theta_{\widetilde{W}^3\,\eta}\Theta_{B\,\zeta}\right) + M_{B1}\cos\theta_W\left(\Theta_{\widetilde{B}\,\zeta}\Theta_{B\,\eta} - \Theta_{\widetilde{B}\,\eta}\Theta_{B\,\zeta}\right)\right],
\end{aligned}
\tag{3.2.23}
$$

with θ_W the Weinberg angle. Similar to our statement concerning the computation of magnetic dipole coupling Ω_0 in the toy model, from (3.2.23) we see that the computation of the magnetic dipole coupling Ω_1 in the MSSM would require analytically cumbersome orders of perturbation theory. But we can bypass such a calculation by deriving a relation between Ω_1 and the pseudo-goldstino mass m_ζ. This calculation for the MSSM proceeds analogously to the calculation for the toy model so we only provide the result here. From the supercurrent coupling to the goldstino we find that coefficient of the magnetic dipole interaction between the goldstino, pseudo-goldstino, and the photon is

$$
\Omega_1 = \frac{m_\zeta}{F_\eta}\left(\Theta_{\widetilde{B}\,\zeta}\cos\theta_W + \Theta_{\widetilde{W}^3\,\zeta}\sin\theta_W\right),
\tag{3.2.24}
$$

where $F_\eta^2 = F_A^2 + F_B^2 + F_u^2 + F_d^2 + (D^3)^2/2 + (D^Y)^2/2$. From the above result we see that as in the case of the toy model, in the MSSM it is easiest to calculate the magnetic dipole coupling *after* we first obtain the mass of the pseudo-goldstino. To compute the mass of the pseudo-goldstino in this case, we must first diagonalize the 4×4 neutralino mass matrix of the pure MSSM and then obtain the eigenstates and eiegenvalues of the non-degenerate spectrum. Such a diagonalization has been done in [16]. After we obtain these eigenstates, we can straightforwardly apply the methods of degenerate perturbation theory to obtain the mass of the pseudo-goldstino. However, the result is not illuminating and is not necessary in the larger scope of the thesis so we will not state it. The result is not specifically necessary because for the case we want to study - the supergravity case - the mass of the pseudo goldstino is already determined to be $2m_{3/2}$ and therefore the calculation of Ω_1 reduces to a calculation of the mixing angles $\Theta_{\widetilde{B}\,\zeta}$ and $\Theta_{\widetilde{W^3}\,\zeta}$. In the next chapter we will derive relations which simplify the evaluation of these mixing angles.

Chapter 4

The Decay in the MSSM with Supergravity

When supersymmetry is promoted from a global to a local symmetry, supergravity results. The essential idea behind supergravity could be explained as follows. Supersymmetry is a symmetry defined by spacetime transformations, and if we make such transformations local, we introduce, due to the algebra of the SUSY generators, diffeomorphism invariance into our theory. Diffeomorphism invariance refers to a theory's invariance under a local change in the coordinate system used to mathematically express the theory. Diffeomorphism invariance is definitive of general theories of gravity, and in such theories where this invariance is connected to a propagating spin-2 field, this field is termed the graviton. However, because we are dealing with a supersymmetric theory, the new spin-2 field must exist in a multiplet ("the gravity multiplet") with other fields of differing spin. Working out the exact theory for supergravity [17], we find there is a spin-3/2 field, termed the gravitino, in this gravity multiplet. The gravitino is important in theories of supergravity because it is, in a sense, the messenger particle of local supersymmetry; in the same way that a photon couples together fields of opposite electric charge, a gravitino couples together fields and their superpartners.

Now, of course, for local supersymmetry to be manifest in nature, it must (just like global supersymmetry) be broken. When a global continuous symmetry is broken, we know from Goldstone's theorem that a massless particle which parameterizes the low energy dynamics of the vacuum state must result. When this breaking occurs in a locally symmetric theory, this goldstone particle becomes the longitudinal component of that symmetry's messenger particle and makes the messenger particle massive. By direct analogy, while the breaking of global supersymmetry results in a massless fermion termed the goldstino, breaking local supersymmetry results in a massive gravitino (i.e., the "messenger particle" of supersymmetry), of which the goldstino is the longitudinal component.

Although we will be studying the effects of supergravity, we will not be using the full machinery of supergravity. Instead we will employ the conformal compensator formalism [18] which isolates the effects of supergravity on matter and gauge fields outside the gravity multiplet. We will use this formalism to derive an important result of multiple local supersymmetry breaking and then extend the result to the MSSM to study its effects on neutralino phenomenology.

The outline of this chapter is as follows. We will study multiple local symmetry breaking in a non-supersymmetric gauge theory to develop intuition for the supersymmetric case. Our focus will be on the effects multiple local symmetries have on the properties of matter particles in the theory. We will then study multiple local SUSY breaking using the conformal compensator formalism to derive the mass relation between gravitinos and goldstinos in such theories. Finally, we will apply this formalism to derive a supergravity corrected neutralino mass matrix and then compute the coupling (3.2.24) of the magnetic dipole interaction for our defining decay process.

4.1 Intuition: Multiple Gauge Symmetry Breaking

Goldstone bosons (i.e, bosons which parameterize the symmetry transformations in a gauge theory) are naturally massless at tree level. However, when the symmetry is broken, these bosons can obtain a mass generated by interactions with the quantum degrees of freedom of the gauge field. We can see this fact most easily through example. We consider a system with two separate sectors of SU(N) gauge symmetry breaking. The lagrangian is

$$\mathcal{L} = (D^\mu h_1)^\dagger D_\mu h_1 + (D^\mu h_2)^\dagger D_\mu h_2 - V(h_1^\dagger h_1) - V(h_2^\dagger h_2), \tag{4.1.1}$$

where h_1 and h_2 are complex scalar fields in the fundamental representation of SU(N),

$$V(h_1^\dagger h_1) = \frac{\lambda}{4}\left(h_1^\dagger h_1 - \frac{v_1^2}{2}\right)^2, \tag{4.1.2}$$

and $V(h_2^\dagger h_2)$ is defined similarly. A standard computation reveals that both h_1 and h_2 acquire VEVs and hence all of the gauge generators are broken. To better elucidate the low energy properties of these fields, we first can rotate the VEVs so that the only exist in the last components of h_1 and h_2 and second, parameterize the dynamical field as an exponential. Doing so we have for h_1

$$h_1 = \frac{v_1}{\sqrt{2}} \exp\left(i\pi_1^A t^A / v_1\right) \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}, \tag{4.1.3}$$

where π_1^A are the goldstone bosons of sector 1, v_1 is the VEV of the gauge field, and t^A and the generators of the gauge group. The field h_2 is similarly parameterized. The above parameterization naively reveals we have two sets of massless goldstone bosons. However, if we include quantum corrections to the classical potential by integrating out the massive gauge field, we find that some of these goldstone bosons acquire a mass. Specifically the 1-loop effective potential for a background field Φ coupled to a gauge field is [19]

$$V_{\text{1-loop}} = \frac{3}{64\pi^2} \mathrm{Tr}\left[M^4(\Phi) \log\left(\frac{M^2(\Phi)}{\mu^2}\right)\right], \tag{4.1.4}$$

where μ^2 is a mass-squared regulator, and $M^2(\Phi)$ is the field dependent mass matrix of the gauge field. For this example $(M^2)^{AB}$ is

$$(M^2)^{AB} = g^2(h_1^\dagger\{t^A, t^B\}h_1 + h_2^\dagger\{t^A, t^B\}h_2), \tag{4.1.5}$$

and upon inserting this mass matrix into (4.1.4) and expanding we find

$$V_{\text{1-loop}} \simeq g^4 v^2 |h_1^\dagger h_2|^2 \log \mu^2 + \cdots . \tag{4.1.6}$$

Substituting the exponential parameterization of h_1 and h_2 and expanding in inverse powers of v_1 and v_2, we find that the above term generates a mass for the axial combination $(v_2\pi_1^A - v_1\pi_2^A)/\sqrt{v_1^2 + v_2^2}$ of goldstone particles while the vector combination $(v_1\pi_1^A + v_2\pi_2^A)/\sqrt{v_1^2 + v_2^2}$ remains massless. In this framework we term the axial combinations the pseudo-goldstone bosons and the vector combinations the actual goldstone bosons. The explanation behind this perturbative mass generation (depicted in Fig. 4-1) was first pro-

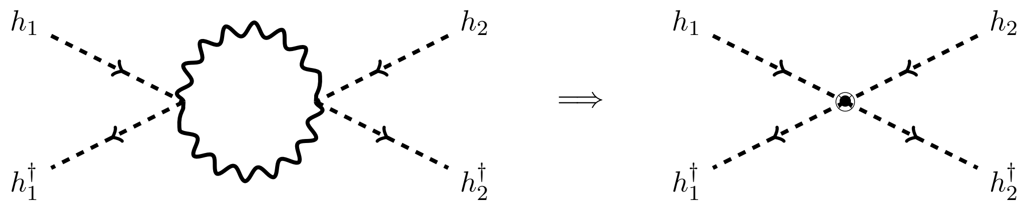

Figure 4-1: Perturbative Mass Generation in a Broken SU(N) Theory: Figure represents a term in the Coleman Weinberg expansion. When the intermediate gauge bosons are integrated out, we generate a mass term for the higgs fields.

vided by Weinberg [20]: The axial SU(N) symmetry is not preserved by our two of symmetry breaking sectors, and so the axial combination of goldstone particles are not associated with any preserved symmetry and hence are not protected against perturbative mass corrections.

The main difference between the gauge symmetry and local supersymmetry cases of multiple breaking, is that in multiple local supersymmetry breaking the pseudo-goldstone particles obtain a mass at tree (and not loop) level. We derive how this occurs in the next section.

4.2 Pseudo-Goldstino – Gravitino Mass Relation

In this section we derive the mass relation between the "pseudo-goldstini" and the gravitino in a theory with multiple local supersymmetry breaking. The derivation closely follows that found in [8]. We will be using the conformal compensator formalism of supergravity in this derivation. In the conformal compensator formalism, the effects of supergravity are encoded in a chiral superfield $\mathbf{\Sigma}$, the so called conformal compensator. We incorporate these effects into our matter theory via the lagrangian

$$\mathcal{L} = -3\int d^4\theta\, \mathbf{\Sigma}^\dagger \mathbf{\Sigma}\, e^{-K(\mathbf{\Phi}^\dagger,\mathbf{\Phi})/3M_{\mathrm{pl}}^2} + \left(\int d^2\theta\, \frac{\mathbf{\Sigma}^3}{M_{\mathrm{pl}}^3} W(\mathbf{\Phi}) + \text{h.c.}\right), \tag{4.2.1}$$

where K is the Kähler potential, W is the superpotential, and M_{pl} is Planck's constant. The component decomposition of $\mathbf{\Sigma}$ is gauge dependent [18], and we will use a form which will allow us to most easily derive our mass result. This form is

$$\mathbf{\Sigma} = M_{\mathrm{pl}} + \theta^2 F_\Sigma, \tag{4.2.2}$$

where F_Σ is the auxiliary field of the conformal compensator.

The derivation of our final mass relation will proceed as follows. We will integrate out F_Σ and then use the result to determine new mass terms for the fields included in the Kähler and superpotential. When we study these new terms in a theory with two independently broken spontaneous symmetries, we find our desired mass relation. First, expanding the exponential in (4.2.1) and using (4.2.2), we have

$$\mathcal{L} = \int d^4\theta \, \boldsymbol{\Sigma}^\dagger \boldsymbol{\Sigma} \left(-3 + \frac{1}{M_{\rm pl}^2} K + \cdots \right) + \int d^2\theta \, \frac{\boldsymbol{\Sigma}^3}{M_{\rm pl}^3} (m_{3/2} M_{\rm pl} + \cdots) \tag{4.2.3}$$

$$= -3F_\Sigma^\dagger F_\Sigma + 3F_\Sigma M_{\rm pl} m_{3/2} + \cdots , \tag{4.2.4}$$

where $m_{3/2}$ is the mass of the gravitino. We obtained the second term in the first line for the superpotential by using the gravitino mass relation $m_{3/2}^2 = |W_0|^2/M_{\rm pl}^4$ where $|W_0|$ is the VEV of the superpotential, and we took the VEV of the Kähler potential to be zero. Integrating out the auxiliary field in (4.2.4) we find $F_\Sigma = m_{3/2} M_{pl}$ and therefore

$$\boldsymbol{\Sigma} = M_{\rm pl}(1 + \theta^2 m_{3/2}). \tag{4.2.5}$$

Now, to understand the implications of this result on systems with multiple local supersymmetry breaking we return to our simplest model of multiple breaking where we have two chiral fields which independently break supersymmetry. This model is defined by the following Kahler potential and superpotential:

$$K(\{\boldsymbol{X}^\dagger, \boldsymbol{X}\}) = \boldsymbol{X}_1^\dagger \boldsymbol{X}_1 + \boldsymbol{X}_2^\dagger \boldsymbol{X}_2, \qquad W(\{\boldsymbol{X}\}) = \gamma_1 \boldsymbol{X}_1 + \gamma_2 \boldsymbol{X}_2, \tag{4.2.6}$$

where $\boldsymbol{X}_1$ has the nonlinear parameterization

$$X_1 = \frac{\eta_1^2}{2F_1} + \sqrt{2}\theta\eta_1 + \theta^2 F_1, \tag{4.2.7}$$

and $\boldsymbol{X}_2$ is defined similarly. We note that F_1 and F_2 have implicitly already been integrated out in this formalism and have the values $-\gamma_1$ and $-\gamma_2$ respectively. The above nonlinear parameterization is important because in a theory with global supersymmetry this parameterization makes the masslessness of the goldstinos η_1 and η_2 manifest. Substituting these results into (4.2.1), expanding the first term, and ignoring the Planck supressed terms in that first term we have

$$\mathcal{L} = \int d^4\theta \, \frac{\boldsymbol{\Sigma}^\dagger \boldsymbol{\Sigma}}{M_{\rm pl}^2} (\boldsymbol{X}_1^\dagger \boldsymbol{X}_1 + \boldsymbol{X}_2^\dagger \boldsymbol{X}_2) + \left(\int d^2\theta \frac{\boldsymbol{\Sigma}^3}{M_{\rm pl}^3} (\gamma_1 \boldsymbol{X}_1 + \gamma_2 \boldsymbol{X}_2) + \text{h.c.} \right). \tag{4.2.8}$$

Next, using the nonlinear parameterization of $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$, (4.2.5), and focusing on the fermion bilinears we find

$$\begin{aligned} \mathcal{L} &= \left(m_{3/2} F_1^\dagger + 3 m_{3/2} \gamma_1 \right) \frac{\eta_1^2}{2F_1} + (1 \to 2) + \text{h.c.} \cdots & (4.2.9) \\ &= -\frac{1}{2} (2 m_{3/2}) (\eta_1^2 + \eta_2^2) + \text{h.c.} + \cdots , & (4.2.10) \end{aligned}$$

where in the second line we used the definition $F_1^\dagger = -\gamma_1$. The last line above is our main result. In it we see that the goldstinos which were massless in global supersymmetry obtain, when we incorporate the effects of supergravity, a mass which is twice that of the gravitino.

We can derive our main result more easily if we explicitly remove the Kahler potential contribution. From (4.2.8) we make the transformation $\boldsymbol{X}_k \to M_{\rm pl} \boldsymbol{\Sigma}^{-1} \boldsymbol{X}_k$. Consequently there is no Kahler potential contribution to the fermion mass and we have simply

$$\begin{aligned} \mathcal{L}_{\rm mass} &= \int d^2\theta \frac{\boldsymbol{\Sigma}^2}{M_{\rm pl}^2} (\gamma_1 \boldsymbol{X}_1 + \gamma_2 \boldsymbol{X}_2) & (4.2.11) \\ &= 2 m_{3/2} \gamma_1 \frac{\eta_1^2}{2F_1} + (1 \to 2) + \text{h.c.} \cdots & (4.2.12) \\ &= -\frac{1}{2} (2 m_{3/2}) (\eta_1^2 + \eta_2^2) + \text{h.c.} + \cdots , & (4.2.13) \end{aligned}$$

which is our main result. This trick of rescaling the fields to eliminate the Kahler potential contribution to the mass will be useful when we consider systems with many fields such as the MSSM.

Our final result is somewhat disconcerting because it suggests something seemingly nonsensical: the longitudinal component of the gravitino, the actual goldstino, appears to have a mass which is larger than the gravitino itself. We see this by noting that the bilinear fermion term in (4.2.10) has a rotational symmetry which allows us to transform η_1 and η_2 into the goldstino and pseudo-goldstino fermions. Defining the pseudo goldstino as $\zeta = (F_2 \eta_1 - F_1 \eta_2)/\sqrt{F_1^2 + F_2^2}$ and the actual goldstino/longitudinal component of the gravitino as $\eta = \widetilde{G}_L = (F_1\eta_1 + F_2\eta_2)/\sqrt{F_1^2 + F_2^2}$ we have

$$\mathcal{L} = -\frac{1}{2}(2m_{3/2})(\zeta^2 + \widetilde{G}_L^2) + \text{h.c.} + \cdots\,, \tag{4.2.14}$$

which suggests that $\widetilde{G}_L$ is heavier than $\widetilde{G}$ itself. However, in a theory of broken local supersymmetry, as in the case of a broken local gauge symmetry, the mass of the goldstone particle is gauge dependent. Therefore although the result above implies that the longitudinal component of the goldstino (i.e., the *true* goldstino) has a mass of $2m_{3/2}$, the actual mass of the goldstino in such a framework is the same as that of the gravitino: $m_{3/2}$. However, we can still take the mass of the *pseudo*-goldstino to be $2m_{3/2}$ because it is uneaten by the gravitino and hence is an independent physical particle without a gauge dependent mass.

In the next section we will see what implications this result has on the phenomenology of neutralinos in the MSSM. Namely, we will extend the model we studied in section 3.2.2 to the realm of local supersymmetry and consider how the decay rate is modified.

4.3 Neutralino Mass Matrix with Two Hidden Sectors and SUGRA

In this section we modify the 6×6 neutralino mass matrix derived in the last chapter to include the effects of supergravity. For this modification we make the assumption that the mass scale which defines supersymmetry breaking in the hidden sector is much larger than the soft mass scales of the original neutralino mass matrix.

We begin with the conformal compensator lagrangian provided in the previous section.

$$\mathcal{L} = -\int d^4\theta \, \frac{\mathbf{\Sigma}^\dagger \mathbf{\Sigma}}{3M_{\text{pl}}^2} \, K(\mathbf{\Phi}^\dagger, \mathbf{\Phi}) + \cdots + \left(\int d^2\theta \, \frac{\mathbf{\Sigma}^3}{M_{\text{pl}}^3} W(\mathbf{\Phi}) + \text{h.c.} \right). \tag{4.3.1}$$

The terms with the gauge superfield strength are not modified by the conformal compensator, so we may ignore them in the calculation of the mass corrections. Moreover, we can simplify the calculation by rescaling all fields according to

$$\Phi \to \frac{M_{\text{pl}}}{\mathbf{\Sigma}} \tag{4.3.2}$$

so that we need not consider the canonical kinetic term contributions. With these considerations we find that the only neutralino mass terms which are changed by the conformal compensator are

$$\delta\mathcal{L}'_{\text{soft}} = -\int d^4\theta\, M_{\text{pl}}^2 \boldsymbol{\Sigma}^{\dagger\,-1}\boldsymbol{\Sigma}^{-1}\frac{\boldsymbol{X}_A^\dagger \boldsymbol{X}_A}{F_A^2}\left[\alpha_{H_u}^2 \boldsymbol{H}_u^{0\,\dagger}\boldsymbol{H}_u^0 + \alpha_{H_d}^2 \boldsymbol{H}_d^{0\,\dagger}\boldsymbol{H}_d^0\right] \\ + \left(\int d^2\theta \frac{\boldsymbol{\Sigma}^2}{M_{\text{pl}}^2}\gamma_A \boldsymbol{X}_A + \text{h.c.}\right) + (A \to B, \alpha \to \beta) + \left(\int d^2\theta\, \frac{\boldsymbol{\Sigma}}{M_{\text{pl}}}\mu\, \boldsymbol{H}_u^0 \boldsymbol{H}_d^0 + \text{h.c.}\right). \tag{4.3.3}$$

Substituting the definition of $\boldsymbol{\Sigma}$, Eq. (4.2.5), and expanding the above lagrangian, we find the only terms proportional to $m_{3/2}$ are

$$\delta\mathcal{L}'_{\text{soft}} = 2m_{3/2}\gamma_A \frac{\eta_A^2}{2F_A} + 2m_{3/2}\gamma_B \frac{\eta_B^2}{2F_B} + \mu m_{3/2} H_u^0 H_d^0 + \cdots . \tag{4.3.4}$$

We consider the last term first. This last term is similar to the b term of the soft mass terms added to the MSSM lagrangian. In the case of two sectors, it simply results in a modification of the sum of the b terms. That is, we have

$$b_A + b_B \to b_A + b_B + \mu m_{3/2}. \tag{4.3.5}$$

Since this is just a rescaling of b_A and b_B, and since $m_{3/2} < m_{\text{soft}}$—as required for the gravitino to be the LSP—we can simply absorb this shift into a redefinition of b_A and b_B.

For the first two terms in (4.3.4), we must be careful about our choices of F_A and F_B. Since there are now additional fields which couple to the hidden super fields we cannot simply replace $F_A = -\gamma_A$ and $F_B = -\gamma_B$. Specifically superpotential which contains hidden sector fields in the MSSM is

$$W = \gamma_A \boldsymbol{X}_A + \gamma_B \boldsymbol{X}_B + \frac{b_A}{F_B}\boldsymbol{X}_A \boldsymbol{H}_u^0 \boldsymbol{H}_d^0 + \frac{b_B}{F_B}\boldsymbol{X}_B \boldsymbol{H}_u^0 \boldsymbol{H}_d^0 + \cdots , \tag{4.3.6}$$

and the F equations are then

$$F_A^\dagger = -\gamma_A - v_u v_d \frac{b_A}{2F_A}, \tag{4.3.7}$$

$$F_B^\dagger = -\gamma_B - v_u v_d \frac{b_B}{2F_B}. \tag{4.3.8}$$

If we were to substitute these values of F_A and F_B into (4.3.4), our simple $2m_{3/2}$ gravitino–pseudo-goldstino mass relation would be modified. However, we recall our limit that the supersymmetry breaking scales of the hidden sectors are much larger than the mass scales of the visible sector. Hence we have $F_A, F_B \gg b_A, b_B$ and we can make the approximation $F_A \simeq -\gamma_A$ and $F_B \simeq -\gamma_B$. So we find that the modifications to the neutralino mass terms due to supergravity effects are

$$\delta\mathcal{L}'_{\text{soft}} \simeq -\frac{1}{2}(2m_{3/2})\left(\eta_A^2 + \eta_B^2\right) + \text{h.c.} \ . \tag{4.3.9}$$

This result is incorporated into the neutralino mass matrix by the simple replacement

$$(M_{\tilde{N}^0})_{AA} \rightarrow (M_{\tilde{N}^0})_{55} + 2m_{3/2}, \tag{4.3.10}$$

$$(M_{\tilde{N}^0})_{BB} \rightarrow (M_{\tilde{N}^0})_{66} + 2m_{3/2}. \tag{4.3.11}$$

4.4 Calculation of Magnetic Dipole Coupling

Our goal is to compute the decay rate for a pseudo-goldstino to go to a goldstino (longitudinal gravitino) plus a photon. The decay occurs through the interaction

$$\mathcal{L} = -\frac{\Omega_1}{\sqrt{2}}\left(\zeta\sigma^{\mu\nu}\eta\right)F_{\mu\nu}, \tag{4.4.1}$$

and has the value

$$\Gamma(\zeta \rightarrow \widetilde{G}_L + \gamma) \simeq \Gamma(\zeta \rightarrow \eta + \gamma) = \frac{m_\zeta^3\,\Omega_1^2}{16\pi}\left(1 - \frac{m_\eta^2}{m_\zeta^2}\right)^3 . \tag{4.4.2}$$

From the conformal compensator formalism we know that the mass of a pseudo-goldstino is $m_\zeta = 2m_{3/2}$ in a supergravity theory with a hidden sector driven supersymmetry breaking scale. We also know that the goldstino, as the longitudinal component of the gravitino, has a mass of $m_\eta = m_{3/2}$. Therefore the only quantity which remains to be determined for this decay rate is the magnetic dipole coupling Ω_1.

In the previous chapter, using the coupling of the supercurrent to the gravitino, we found a theoretical expression Ω_1

$$\Omega_1 = \frac{m_\zeta}{F_\eta}\left(\Theta_{\widetilde{B}\,\zeta}\cos\theta_W + \Theta_{\widetilde{W}^3\,\zeta}\sin\theta_W\right). \tag{4.4.3}$$

Proceeding as we did in the toy model, we can compute each of the mixing angles using perturbation theory and obtain a closed form expression for Ω_1. However, due to the necessity of having a diagonalized the zeroth-order matrix in perturbation theory, the calculation does not immediately yield a clean result For example, if we computed $\Theta_{\widetilde{B}\,\zeta}$ using perturbation theory we would obtain

$$\Theta_{\widetilde{B}\,\zeta} = \langle \widetilde{B}|\zeta\rangle \tag{4.4.4}$$

$$= \langle \widetilde{B}|\zeta^{(0)}\rangle + \sum_{k=1}^{4}\langle \widetilde{B}|k\rangle \frac{\langle k|\Delta_1|\zeta^{(0)}\rangle}{m_\zeta^{(0)} - m_k^{(0)}} + \cdots\,, \tag{4.4.5}$$

where

$$\Delta_1 = M^{6\times 6}_{\widetilde{N}^0} - \lim_{F_A,F_B\to\infty} M^{6\times 6}_{\widetilde{N}^0}, \tag{4.4.6}$$

and $|k\rangle$ are the non-degenerate eigenstates of $\Delta_0 = \lim_{F_A,F_B\to\infty} M^{6\times 6}_{\widetilde{N}^0}$. These states $|k\rangle$ are simply the mass eigenstate neutralinos of the original MSSM neutralino mass matrix and m_k^0 are their corresponding masses. It is possible to obtain this part of the eigenspectrum [16] but the result will be opaque to intuition. Instead it will prove useful to derive a formula which simplifies the calculation.

First, we make take the approximation $m_{3/2} \ll m_k^0$ for all k. This is not an unfounded assumption because for the gravitino to be the LSP we must have $m_{3/2} < m_{\rm soft}$ where $m_{\rm soft}$ is a typical soft-SUSY breaking or visible sector scale. We can then incorporate corrections to this limit via a power series. Next, we note that the eigenkets $|\widetilde{B}\rangle = (1,0,0,0,0,0)$ and $|\zeta^{(0)}\rangle = (0,0,0,0,F_B,-F_A)/F_{\rm Eff}$ are orthogonal. With these considerations the mixing angle reduces to

$$\Theta_{\widetilde{B}\,\zeta} = -\sum_{k=1}^{4}\langle \widetilde{B}|k\rangle \frac{\langle k|\Delta_1|\zeta^{(0)}\rangle}{m_k^{(0)}} + \cdots \tag{4.4.7}$$

Next, we use the fact that $\langle k|M_0^{-1} = \langle k|(m_k^{(0)})^{-1}$ to write

$$\Theta_{\widetilde{B}\,\zeta} = -\sum_{k=1}^{4}\langle \widetilde{B}|k\rangle\langle k|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle + \cdots\,. \tag{4.4.8}$$

The sum above does not run over a complete set of states for the 6×6 matrix, but we can insert two terms which complete the set. Using the fact that the bino state is orthogonal to both the zeroth order pseudo-goldstino state and the zeroth order goldstino state $|\eta^{(0)}\rangle = (0,0,0,0,F_A,F_B)/F_{\text{Eff}}$, we may write

$$\begin{aligned}\Theta_{\widetilde{B}\,\zeta} = &-\sum_{k=1}^{4}\langle\widetilde{B}|k\rangle\langle k|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle - \langle\widetilde{B}|\zeta^{(0)}\rangle\langle\zeta^{(0)}|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle \\ &- \langle\widetilde{B}|\eta^{(0)}\rangle\langle\eta^{(0)}|\Delta_0^{-1}\Delta_1|\eta^{(0)}\rangle + \cdots \qquad (4.4.9)\\ = &-\langle\widetilde{B}|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle + \cdots . \qquad (4.4.10)\end{aligned}$$

where in the last line we used the completeness of states to eliminate the internal ket-bra terms. Now, performing a similar reduction on $\Theta_{\widetilde{W}^3\,\zeta}$ and setting $m_\zeta \simeq 2m_{3/2}$ from (4.4.3) we find

$$\Omega_1 = -\frac{2m_{3/2}}{F_\eta}\left(\langle\widetilde{B}|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle\cos\theta_W + \langle\widetilde{W}^3|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle\sin\theta_W\right) + O(m_{3/2}^2). \qquad (4.4.11)$$

It is now procedural matter to compute this quantity. For simplicity we define the dimensionless quantity in the parentheses Θ_{eff}. Using *Mathematica* we find (to first order in the D terms)

$$\begin{aligned}\Theta_{\text{eff}} = &\left(\langle\widetilde{B}|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle\cos\theta_W + \langle\widetilde{W}^3|\Delta_0^{-1}\Delta_1|\zeta^{(0)}\rangle\sin\theta_W\right) + O(m_{3/2}) \qquad (4.4.12)\\ = &-\frac{e}{\Lambda_1^5}(M_1 - M_2)\left[\frac{F_A}{F_B}\Big(b_B(v_d^2 - v_u^2) + v_d v_u(\beta_{H_d}^2 - \beta_{H_u}^2)\Big)\right. \qquad (4.4.13)\\ &\left.+\frac{F_B}{F_A}\Big(b_A(v_u^2 - v_d^2) + v_d v_u(\alpha_{H_u}^2 - \alpha_{H_d}^2)\Big)\right]\\ &-\frac{\sqrt{2}e}{\Lambda_1^5}\left(\frac{D^3}{g}\right)\left(\frac{F_B}{F_A}M_{A2} - \frac{F_A}{F_B}M_{B2}\right)\left[(g^2 + g_Y^2)v_u v_d - \mu M_1\right]\\ &-\frac{\sqrt{2}e}{\Lambda_1^5}\left(\frac{D^Y}{g_Y}\right)\left(\frac{F_A}{F_B}M_{B1} - \frac{F_B}{F_A}M_{A1}\right)\left[(g^2 + g_Y^2)v_u v_d - \mu M_2\right]\\ &+\mathcal{O}(m_{3/2}; (D^Y)^2; (D^3)^2) \qquad (4.4.14)\end{aligned}$$

where

$$\Lambda_1^5 = F_\eta\left[(g^2 M_1 + g_Y^2 M_2)v_u v_d - \mu M_1 M_2\right], \qquad (4.4.15)$$

$e = g \sin\theta_W$ is the electromagnetic coupling. With this result the decay rate is completely determined. We note that this mixing angle mirrors the behavior of the coefficient Ω_0 in the toy model in that it vanishes when each hidden sector couples identically to the visible sector and has the same supersymmetry breaking scale.

We can use this result and the expected order of magnitude energy scales for the parameters in the hidden and visible sectors to estimate the lifetime for this process. From Eq. (4.4.2) the decay rate is

$$\Gamma(\zeta \to \widetilde{G}_L + \gamma) \approx (2m_{3/2})^3 \left(\frac{2m_{3/2}\Theta_{\rm eff}}{F_\eta}\right)^2 \times 10^{-2} \approx \frac{m_{3/2}^5 \Theta_{\rm eff}^2}{F_\eta^2} \times 10^{-1}. \qquad (4.4.16)$$

Taking $m_{3/2} \simeq 100$ GeV, $F_\eta \sim (10^{10}\ \text{GeV})^2$ [7], and the mass parameters of each hidden sector to be roughly of the same order of magnitude, i.e., $M_{A2}/M_{A1} \sim O(1)$, etc. We find that $\Theta_{\rm Eff} \sim O(10^{-17})$ and the lifetime of the goldstino (computed by reintroducing $\hbar$ to change the energy scale to a time scale) is

$$\tau \sim 10^{39}\ \text{sec} \left(\frac{10^{-17}}{\Theta_{\rm eff}}\right)^2 \left(\frac{100\ \text{GeV}}{m_{3/2}}\right)^5 \left(\frac{F_\eta}{10^{20}\ \text{GeV}^2}\right)^2, \qquad (4.4.17)$$

which is much longer than the age of the universe ($\sim 10^{17}$ sec). We note this lifetime corresponds only to a specific region of the hidden sector parameter space in which the sectors have similar couplings and breaking scales; other regions of the parameter space will yield different lifetimes.

Chapter 5

Discussion

In this thesis we began by assuming there are two independent hidden sectors—as opposed to the customary, one hidden sector—responsible for supersymmetry breaking. The main consequence of this assumption is that a low mass pseudo-goldstino is added to the spectrum of neutralinos and becomes the next-to-lightest supersymmetric particle. Moreover, when this assumption is studied in the context of supergravity, one finds that—assuming a relatively large mass scale for the hidden sectors—the pseudo-goldstino acquires a mass which is twice that of the gravitino [8] . This mass ratio has interesting implications on collider signatures [21], but also has cosmological significance and the purpose of this thesis was to investigate a decay scenario which would clarify the implications this mass ratio result has on cosmology. In particular we derived an analytic formula of the rate for a pseudo-goldstino to decay to a gravitino plus a photon through a magnetic dipole interaction. Depending on the lifetime implied by this decay scenario, and hence dependent on the parameters which define the coupling between the two hidden sectors and the visible sector, the pseudo-goldstino could be could be inconsistent with standard cosmology [22]. A future investigation would consider which areas of parameter space are allowed given cosmological constraints.

Also according to (4.4.14), if the two hidden sectors are identically coupled to the visible sector and have the same supersymmetry breaking scales then the magnetic dipole coupling vanishes. We have yet to determine the physical meaning of this behavior and hope to better understand it a subsequent project.

There are various ways one can extend the results in this thesis. One straightforward way is to take the assumption of multiple breaking a step further and assume there are three (or more) sectors of supersymmetry breaking. This would result in a collection of roughly degenerate "pseudo-goldstini" [8] all of which have a mass twice the gravitino mass. The cosmological implications of such a scenario have yet to be rigorously investigated and the decay processes resulting from it can be the topic of a future investigation.

Finally, the main decay may also have relevance to dark matter phenomenology. A recent experimental analysis of data from the Fermi Large Area Telescope found a gamma ray peak centered at around 130 GeV coming from the center of our galaxy [23]. If we interpret this peak as arising from the decay of dark matter particles we can find a concrete application for the model studied in this thesis. Taking dark matter to be composed of both pseudo-goldstino and gravitino particles, we can then take the source of this peak to be the result of a pseudo-goldstino emitting a photon in a decay to a gravitino. When we consider this interpretation within the framework of supergravity, we find - from a simple kinematics calculation - that $E_\gamma = \frac{3}{4}m_{3/2}$ and hence in order to produce a 130 GeV peak the mass of the gravitino must be $m_{3/2} \simeq 173.3$ GeV. This result for $m_{3/2}$ is within the limits expected for the consistency with the standard properties of gravity mediated supersymmetry breaking, and hence makes the model studied in the thesis deserving of further study. In the last chapter, we computed the lifetime of this process for a certain region of the hidden sector-visible sector coupling parameter space, finding that the result was much larger than the age of the universe. This result can be used to constrain the values of the parameters in this model and it would be worthwhile to obtain more stringent constraints by ensuring the resulting decay rate is consistent with the production rate of photons from the galactic center.

Bibliography

[1] S. P. Martin, "A Supersymmetry primer," 1997.

[2] M. Gonzalez-Garcia and Y. Nir, "Neutrino masses and mixing: Evidence and implications," *Rev.Mod.Phys.*, vol. 75, pp. 345–402, 2003.

[3] S. E. Rugh and H. Zinkernagel, "The quantum vacuum and the cosmological constant problem," *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics*, vol. 33, no. 4, pp. 663–705, 2002.

[4] M. E. Peskin, "Dark matter and particle physics," *J.Phys.Soc.Jap.*, vol. 76, p. 111017, 2007.

[5] J. L. Rosner, "Resource letter: The Standard model and beyond," *Am.J.Phys.*, vol. 71, pp. 302–318, 2003.

[6] J. Ellis and K. A. Olive, "Supersymmetric Dark Matter Candidates," 2010.

[7] J. Terning, *Modern Supersymmetry.* Oxford University Press, first ed., 2006.

[8] C. Cheung, Y. Nomura, and J. Thaler, "Goldstini," *JHEP*, vol. 1003, p. 073, 2010.

[9] J. Wess and B. Zumino, "Supergauge transformations in four dimensions," *Nuclear Physics B*, vol. 70, no. 1, pp. 39–50, 1974.

[10] M. Drees, R. M. Godbole, and P. Roy, *Theory and phenomenology of sparticles*, vol. 1. 2004.

[11] Z. Komargodski and N. Seiberg, "From linear susy to constrained superfields," *Journal of High Energy Physics*, vol. 2009, no. 09, p. 066, 2009.

[12] L. Girardello and M. T. Grisaru, "Soft breaking of supersymmetry," *Nuclear Physics B*, vol. 194, no. 1, pp. 65–76, 1982.

[13] C. Csaki, "The minimal supersymmetric standard model (mssm)," *arXiv preprint hep-ph/9606414*, 1996.

[14] P. Binetruy, *Supersymmetry: Theory, Experiment, and Cosmology.* Oxford University Press, first ed., 2006.

[15] K. Gottfried and T.-M. Yan, *Quantum mechanics: fundamentals.* Springer, 2003.

[16] S. Choi, J. Kalinowski, G. Moortgat-Pick, and P. Zerwas, "Analysis of the neutralino system in supersymmetric theories," *The European Physical Journal C-Particles and Fields*, vol. 22, no. 3, pp. 563–579, 2001.

[17] J. Wess and J. Bagger, *Supersymmetry and supergravity.* Princeton University Press, 1992.

[18] Cheung, Clifford and D Eramo, Francesco and Thaler, Jesse, “Supergravity computations without gravity complications,” *Physical Review D*, vol. 84, no. 8, p. 085012, 2011.

[19] S. Coleman and E. Weinberg, “Radiative corrections as the origin of spontaneous symmetry breaking,” *Physical Review D*, vol. 7, no. 6, p. 1888, 1973.

[20] Weinberg, Steven, “Approximate symmetries and pseudo-Goldstone bosons,” *Physical Review Letters*, vol. 29, no. 25, pp. 1698–1701, 1972.

[21] C. Cheung, J. Mardon, Y. Nomura, and J. Thaler, “A Definitive Signal of Multiple Supersymmetry Breaking,” *JHEP*, vol. 1007, p. 035, 2010.

[22] D. Bertolini, K. Rehermann, and J. Thaler, “Visible Supersymmetry Breaking and an Invisible Higgs,” *JHEP*, vol. 1204, p. 130, 2012.

[23] C. Weniger, “A Tentative Gamma-Ray Line from Dark Matter Annihilation at the Fermi Large Area Telescope,” *JCAP*, vol. 1208, p. 007, 2012.