Problem 2. Motion in one-dimension and two-dimensions

(a)

Solution: Before we can begin drawing our graphs, we must realize that we do not have all the information we need. For example, we know that the ant moves with a velocity +2 m/s for 2 min in the second segment of its motion, but we do not know where it ends up when it changes its motion. We must solve for this type of information before we can graph anything. First, we will list the time intervals, both known and unknown, which define a unique motion for the ant.

The ANT Begins at $t_0 = 0$ min and remains there until $t_1 = 10$ min (For $\Delta t_1 = t_1 - t_0 = 10$ min the ant is stationary) Then for a time Δt_2 later it is moving at a speed 2 m/s. This time $\Delta t_2 = 2$ min. Next for some unknown time interval Δt_3 the ant slows its speed down to 0.5 m/s. From that point it breaks to a halt for $\Delta t_4 = 20$ min. Then it moves backwards at a constant speed for some unknown time Δt_5. And lastly, it breaks to a halt during some time Δt_6

Collecting all this information, we have

$$\Delta t_1 = 10 \text{ min} = 600 \text{ s}$$
$$\Delta t_2 = 2 \text{ min} = 120 \text{ s}$$
$$\Delta t_3 = ??$$
$$\Delta t_4 = 20 \text{ min} = 1200 \text{ s}$$
$$\Delta t_5 = ??$$
$$\Delta t_6 = ??$$

where the "??" label time intervals we must solve for.

Each time interval defines a specific motion of the ant and therefore defines the ants position. For example, in the time interval Δt_1 the ant begins at $x_0 = 0$ cm and ends at $x_1 = 0$ cm because it does not move during this time.

Similarly for Δt_2

- ANT Begins at $x_1 = 0$ cm and ends at x_2, where

$$x_2 = x_1 + v_1 \Delta t_2$$

From the problem statement $v_1 = 2$ cm/s & $\Delta t_2 = 120$ s so

$$x_2 = 0\text{ cm} + 2\text{ cm/s} \times 120\text{ s}$$
$$= 240\text{ cm}$$

For Δt_3

- ANT Begins at $x_2 = 240$ cm and ends at x_3, where

$$x_3 = x_2 + v_1 \Delta t_3 + \frac{1}{2} a_1 \Delta t_3^2$$

$$0.5 = v_1 + a_1 \Delta t_3$$

where $v_1 = 2$ cm/s, $a_1 = -0.1$ cm/s^2, and Δt_3 is the time it takes the ant to reach a speed of 0.5 cm/s

Using the 2nd eqn to solve for this time, we have

$$0.5\text{ cm/s} = 2\text{ cm/s} - 0.1\text{ cm/s}^2\, \Delta t_3$$
$$\Downarrow$$
$$\Delta t_3 = 15\text{ s}$$

AND THEREFORE

$$x_3 = 240\,cm + 2\,cm/s\,(15\,s) - \frac{0.1\,cm/s^2}{2}(15\,s)^2$$

$$= 240\,cm + 30\,cm - 0.05\,cm/s^2 \cdot 225\,s^2$$

$$= 270\,cm - 11.25\,cm = \boxed{258.75\,cm}$$

For Δt_4

- ANT is stationary so

$$x_4 = x_3$$

For Δt_5

- ANT is moving ~~backwards~~ at a ~~speed~~ velocity $v_2 = -2\,cm/s$ for a distance of 240 cm. So

$$240\,cm = |x_5 - x_4| = |v_2 \Delta t_5|$$

so

$$\boxed{\Delta t_5 = 120\,s}$$

AND

$$x_5 = x_4 + v_2 \Delta t_5$$

$$= 258.75\,cm - 240\,cm = \boxed{18.75}$$

- For Δt_6

- ANT is deaccelerating in the negative direction (accelerating in the positive direction) with $a_2 = 0.5\,cm/s^2$ until it stops so

$$0 = v_2 + a_2 \Delta t_6$$

$$= -2\,cm/s + 0.5\,\frac{cm}{s^2}\,\Delta t_6 \Rightarrow \boxed{\Delta t_6 = 4\,s}$$

AND

$$x_6 = x_5 + v_2 \Delta t_6 + \frac{1}{2} a_2 \Delta t_6^2$$

$$= 18.75 + (-2\,cm/s)(4\,s) + \frac{0.5\,cm/s^2}{2}(4\,s)^2$$

$$x_6 = 18.75 - 8\,cm + 4\,cm$$

$$= 14.75\,cm$$

Collecting all of this information, we find

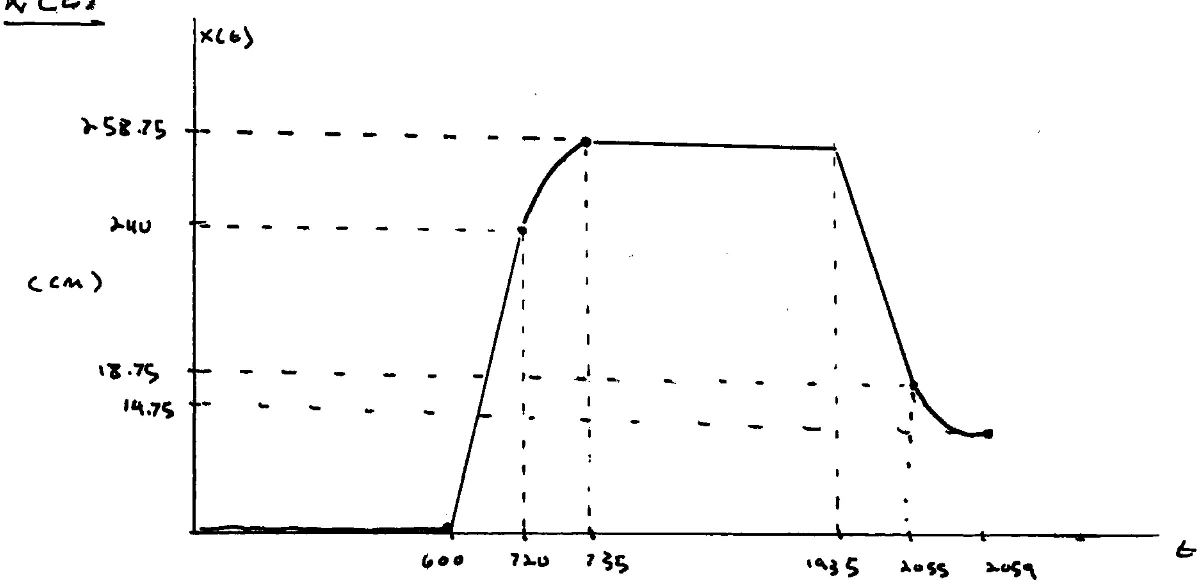
Positions	Time Between Positions	Position Behavior For Interval
$x_0 = 0$		
	$\Delta t_1 = 600\,s$	$\rightarrow x(t) = 0$
$x_1 = 0$		
	$\Delta t_2 = 120\,s$	$\rightarrow x(t) = v_1 t$
$x_2 = 240\,cm$		
	$\Delta t_3 = 15\,s$	$\rightarrow x(t) = x_2 + v_1 t + \frac{1}{2} a_1 t^2$
$x_3 = 258.75\,cm$		
	$\Delta t_4 = 1{,}200\,s$	$\rightarrow x(t) = x_3$
$x_4 = 258.75\,cm$		
	$\Delta t_5 = 120\,s$	$\rightarrow x(t) = v_2 t$
$x_5 = 18.75\,cm$		
	$\Delta t_6 = 4\,s$	$\rightarrow x(t) = x_5 + v_2 t + \frac{1}{2} a_2 t^2$
$x_6 = 14.75\,cm$		

Where $v_1 = 2\,cm/s \qquad v_2 = -2\,cm/s$

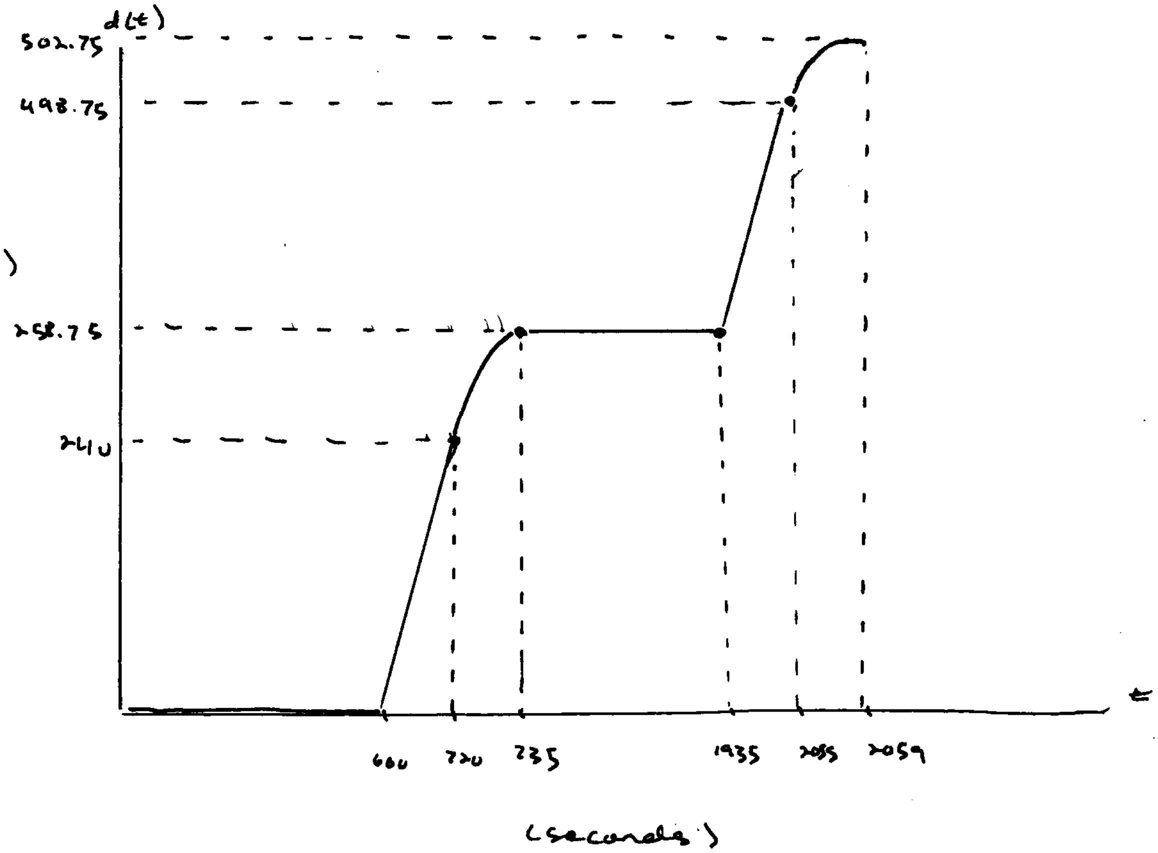
$a_1 = -0.1\,cm/s^2 \qquad a_2 = 0.5\,cm/s^2$

WITH THIS INFO, WE CAN FINALLY MAKE A (NOT TO SCALE) ~~GRAPH~~ GRAPH

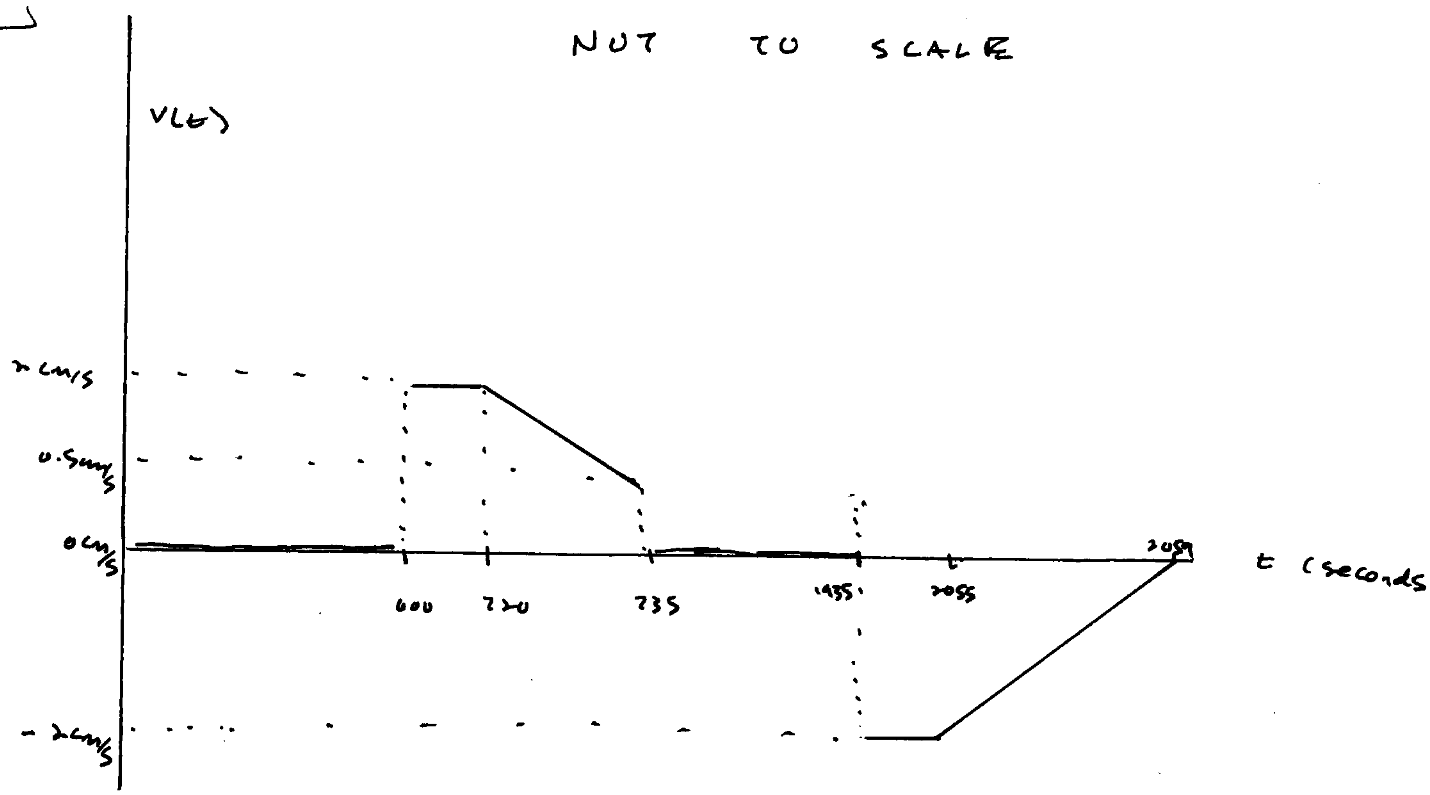
$\underline{x(t)}$



$d(t)$: Defined as total distance traveled



$V(t)$:



Problem 2. c. Graph