Large Numbers and Emergence

These notes[1] are part of a series concerning "Motifs in Physics" in which we highlight recurrent concepts, techniques, and ways of understanding in physics. In these notes we discuss Philip Anderson's idea of more degrees of freedom yielding qualitatively properties from those of a single degree of freedom, and we show how this idea is related to the novel 21st century notion of "emergent properties".

The Small and the Many

In 1986, historian and particle physicist Abraham Pais wrote *Inward Bound* [1], a chronicle of mankind's efforts in the 20th century to understand the building blocks of matter. As the title suggests, Pais's focus was mostly on the particle content of our modern theories of physics, phenomena like the beta rays which were later discerned to be electrons and even later discerned to be part of the theory of quantum electrodynamics. Another way to state this focus is to say Pais was concerned with his century's most fundamental descriptions of matter.

The word "fundamental" here deserves clarification. By "fundamental" we do not mean the qualitative notion of importance or significance, but rather the specifically physical definition in which theory A is "more fundamental" than theory B if theory A is defined at a higher energy scale or smaller length scale than theory B. Thus, the quantum mechanics of particles is more fundamental than the classical mechanics of particles; quantum electrodynamics is more fundamental than quantum mechanics; and string theory (God willing) is more fundamental than quantum electrodynamics.

Despite this rather specific definition, it is often easy to conflate the physical meaning of "fundamental" with its meaning in everyday contexts. Indeed this was the habit of many 20th century particle physicists who believed that since they were working at the highest energy limits of physics, their work was unquestionably more important than the work of their colleagues. For example, Murray Gell-Mann the physicist responsible for organizing the particle content of nuclear physics, once labeled the work of condensed matter physicists like John Bardeen and Philip Anderson as "Squalid-State Physics" [2]. In *Inward Bound* Pais never seemed to make such disparaging references, but the preoccupations of his historical work [3, 4, 5] suggests that he, like many of his fellow particle physicists, believed that most important discoveries of the past century concerned physics at the smallest length scales.

Years earlier this conviction was pre-emptively challenged by Philip Anderson himself in the now famous article "More is Different" [6]. In the article, Anderson argued that our understanding of matter in aggregate is not reducible to our understanding of matter in singular, and that knowledge of the fundamental physical laws of the universe does not, in turn, give us knowledge of all of science. As he puts it,

> . . . the more the elementary particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

As a simple example, consider the relationship between quantum mechanics and biological molecules. Simply because physicists have developed a very accurate theory of electrons and other subatomic particles does not directly imply that we can understand how or why those particles come together to create the molecular constituents of life (e.g., DNA, proteins, and mRNA). In other words, understanding the most fundamental laws of physics does not supply us with a complete understanding of the physical world around us for when those laws are manifest in complex many particle system we observe properties which do not seem to directly follow from the laws themselves.

It is difficult and perhaps unimportant to say which focus, Pais's or Anderson's, has won out. High energy physicists continue to work in the rarefied mathematical realms of the most fundamental models of matter

[1] Inspired by a blog post by Philip Tanedo

and energy, but many other fields of science have taken Anderson's thoughts and expanded them into a more general idea termed "emergence" [7]. Emergence means various things to physicists and philosophers but for our discussion we will take it to mean "properties which are present in macrocosm but absent in the corresponding microcosm[2]."

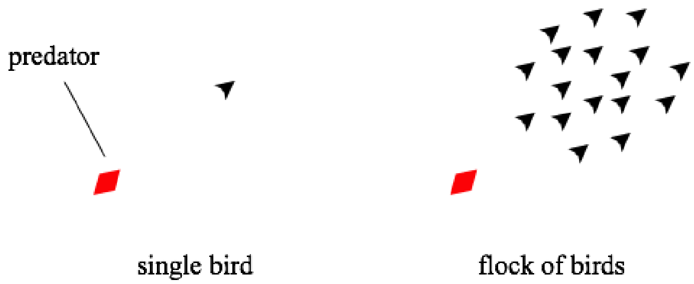

Figure 1: In the simplest computational flocking models [8], the behavior of a single bird in the presence of a predator is qualitatively different from the behavior of a flock of birds in the presence of the same predator.

One everyday example is flocking in some bird species in which large numbers of birds can appear to move like a single organism whose style and properties of motion are quite different from the style and properties of motion of a single bird. This behavior can be simulated so that local rules of interaction between birds leads to the global behavior observed in flocks [8]. In this example, the flocking behavior would be said to "emerge" from the aggregation of many locally communicating birds.

Types of Emergence

In these notes, we will focus on the emergent properties associated with transitioning from $N = 1$, or single degree-of-freedom (i.e., d.o.f., our general term for a particle, spin, or body in a system) to large N, or many degree-of-freedom systems[3]. The way emergence connects the single to the many can be variously organized according to how explicit the relationship is between these two types of systems. Sometimes the relationship is such that the large d.o.f. theory can be directly derived from the single d.o.f. theory. Other times, even when there is no clean analytic mapping from the single d.o.f. theory to the many d.o.f. theory, there is still an accepted physical model which well describes the latter. Below, we discuss these two types of emergence respectively termed deducible and non-deducible emergence[4]

> **Why is the distinction important?:**
>
> It is necessary to distinguish between a deducible emergence and a non-deducible emergence because each one requires a different type of physical modeling to describe the large N phenomena. A system with a deducible emergence can be studied through a reductionist framing in which we obtain knowledge of the whole system by analyzing its constituent parts. A system with a non-deducible emergence requires new theoretical and conceptual models not only to understand the large N system but also to understand its

[2]Sometimes scientists define emergence more widely to include the transition from a "fundamental theory" to an "effective theory", e.g., classical mechanics "emerging" from quantum mechanics. In order to not repeat elements of the previous discussion on effective theories, we will define emergence only according to the differing physical properties observed for a small number of particles versus. those observed in a large number of particles.

[3]How large is "large"? It depends. Chaos can be said to emerge from Newtonian dynamics once we have three or more gravitating particles , but we need some arbitrary $N \gg 1$ number of particles before the results of statistical mechanics are applicable [9].

[4]These categories are inspired by, but are not identical to, the work of D. Chalmers [10]. Specifically, his categories of weak and strong emergence do not map onto our categories of deducible and non-deducible emergence. For example, we will consider many examples of non-deducible emergence, but Chalmers believes consciousness is the only example of strong. emergence.

connection to its constituents. Thus, the latter represents a more radical departure from existing physical theories and thus supplements our understanding of the phenomena by requiring new physical theories and auxiliary ideas to describe it.

Of course, the labeling of a phenomena as a deducible or a non-deducible emergence is most useful if it is accomplished before we attempt to build any physical models. However, it is not always possible to determine *a priori* in which category a macroscopic physical system falls into. In practice, one reliable clue that a system exhibits non-deducible emergence is that naive combinations of the constituent parts do not lead to the macroscopic phenomena we are attempting to understand.

- **Deducible Emergence:**

  We say that a large N system A has a deducible emergence with respect to a small N system B, if it is possible to obtain the properties of system A by analytically deriving or computationally modeling the large N limit of system B. This degree of emergence is perhaps the most intellectually satisfying of two we discuss, for although the emergent system has qualitative properties different from those of the single degree of freedom system, it is at least clear how the theoretical structure of the latter leads to the properties of the former.

  The Central Limit theorem[5] is an important result in probability theory which mirrors the spirit of deducible emergence. This theorem states that N independent random variables $x_1, \ldots, x_N$ which are all distributed according to the same probability density function p, and which all have a finite mean $\langle x_1 \rangle$ and variance $\text{Var}(x_1)$, always result in the the random variable $X = \sum_{i=1}^{N} x_i$ having (in the $N \gg 1$ limit) the the probability density function

$$p(X) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \exp\left[-\frac{(X - \overline{X})^2}{2\sigma_X^2}\right] \tag{1}$$

  where $\overline{X} = N\langle x_1 \rangle$ and $\sigma_X^2 = N\text{Var}(x_1)$. In this result, the gaussian probability density function of the aggregate random variable X is independent of the underlying distribution of x_i. Thus, we could say that the gaussian distribution is "deductively emergent" from the independent and identically distributed random variables.

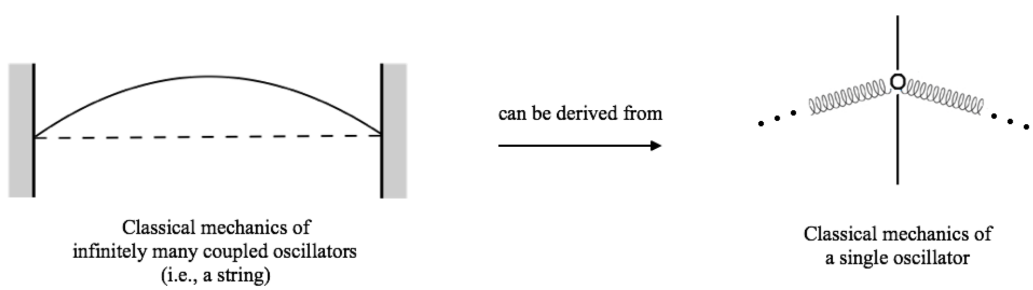

  Figure 2: In deducible emergence, there is an accepted theory for the large N system and this theory can be derived from the theory of the small N system. The dynamical equations of continuum mechanics (e.g., the wave equation of a string) can typically be derived from the classical mechanics of the individual particles which make up the system.

  Other examples of deducible emergence are the relationship between the classical mechanics of continua and the classical mechanics of single particles (e.g., the wave equation and the equation for a

[5] There are variants of the theorem. We will actually only discuss the simplest version of the theorem.

simple harmonic oscillator); the relationship between the behavior of flocks of bird-like objects and the behavior of a few of those birds [8]; the relationship between the nuclear particle masses computed in quantum chromodynamics and the quantum chromodynamics of quarks and gluons.

- **Non-deducible Emergence:**

  We say that a large N system A has a non-deducible emergence with respect to a small N system B if it is *not* possible to derive the properties of system A by simply analyzing multiple copies of A. This definition is best understood through an example: Statistical mechanics has a non-deducible emergence with respect to classical mechanics because statistical mechanics is an accepted theory of $N \gg 1$ particle systems, but it cannot be derived from single-particle classical mechanics. Moreover, statistical mechanics provides a mathematical framework to understand **phase transitions**, abrupt qualitative changes in the properties of a macroscopic system which cannot be understood from a single d.o.f. analysis.

  Historically, his type of emergence has been the most difficult to discover because physicists have often operated under a reductionist mindset in which the properties of a complicated system are assumed to be reducible to the properties of its components. That this is not always true came to be known through the development of statistical mechanics. The onset of chaotic behavior whenever we move from single to many particle systems means that the precise equations of classical dynamics are no longer tractable and we thus must replace precision with probability, or classical with statistical mechanics (Fig. 3). However, although statistical mechanics cannot be derived from classical mechanics and although it contains auxiliary ideas like entropy and phase transitions which have no classical mechanics analog, it is still defined by physical laws (principally conservation of energy) which are consistent with the laws of classical mechanics.

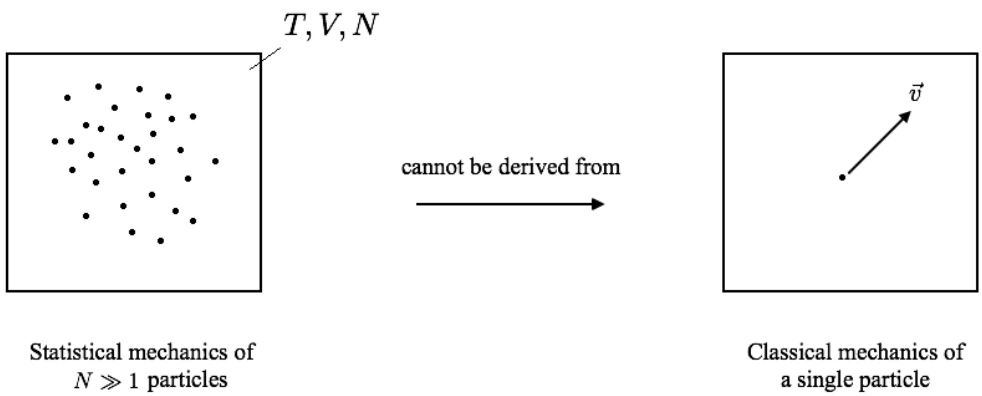

Figure 3: In non-deducible emergence, the properties of the large N system cannot be derived from the properties of the small N system. For the case of statistical mechanics, the onset of chaos makes it analytically intractable to study the classical dynamics of many particle systems. Thus the precision of classical mechanics is replaced by the probabilities of statistical mechanics.

Other examples of non-deducible emergence include the relationship between BCS superconductivity and quantum mechanics and electrodynamics; the relationship between superfluidity and fluid dynamics and quantum mechanics; the relationship between protein structure/function and amino acid sequence Fig. 4.

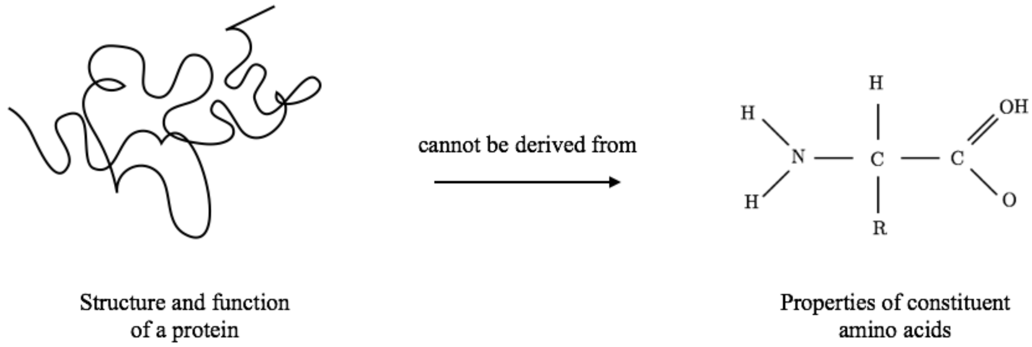

Figure 4: Proteins are a non-deductively emergent system with respect to their constituent amino acids. This is to say that we cannot, from first principles, predict the structure and function of a protein simply from a knowledge of its chain of amino acids. (Being able to do so in all cases would amount to a full solution of the **protein folding problem.**) There are methods to make this prediction, but they are grounded in heuristic techniques rather than analytic methods of the kind which show how the properties of a continuous string can be derived from the properties of the individual oscillators which compose it.

Biology as Emergent Phenomenon

Life is one of the classic examples of a non-deducible emergence. Biological physics is concerned with traversing the divide between the animate and the inanimate and it seeks out physical explanations for various aspects of life and life processes. Because the distance, at the level of theoretical modeling, between these physical laws and the unicellular life[6] which is constrained by them is so vast, there are various ways to "do" biological physics.

Some scientists attempt to simulate observable characteristics of life and thus try to understand how local rules give rise to non-local and complex behavior [11, 12]. Other mathematically inclined scientists approach the study biology in the same way early physicists proverbially approached the study of the inanimate world, that is they use observable phenomena as a starting point and from those phenomena they attempt to abstract mathematically rendered principles to explain new phenomena [13, 14, 15]. In such schemes the mathematical principles which define the biological system are as divorced from the underlying physics of the system as the mathematical methods of finance are divorced from the political and economic theory which gives these methods their relevance. Still there are other scientists (almost always physicists) who are uncomfortable with any possible division between physical laws and biological laws. These scientists seek new mathematical principles by which to understand biological phenomena, but these principles always extend from the physics we know must underlie the system [16, 17].

The reason for all these diverging attempts to try to understand the principles (physical or otherwise) of biology is that the emergent relationship between, say, unicellular life and chemistry seems to be qualitatively different from the emergent relationship between statistical mechanics and classical mechanics (See Fig. 5). While the latter relationship can be confidently characterized as remaining within the domain of physics, the former is characterized by a transition from reducible physical systems to something else.

Therefore, trying to use mathematics to understand biology in the same way we use math to understand physics is difficult because unlike the clean idealized systems of physics, the basic building blocks of life contain many different macromolecules each of which have specific properties and functions and each of which are composed of hundreds or thousands of their own molecules. It almost seems as if to really solve the problem carefully would require separate but connected theories for each unique organelle and macro-

[6] For simplicity and specificity, we will take unicellular life as a proxy for all biological systems which are related to it.

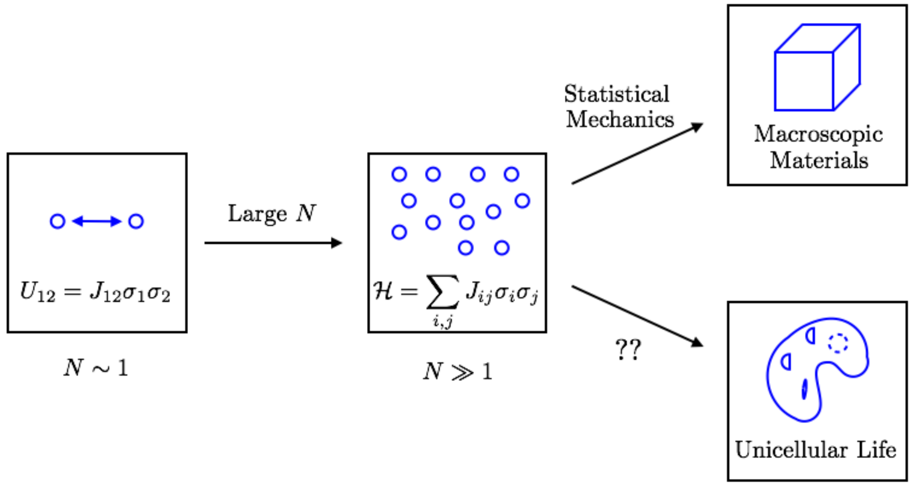

Figure 5: Many body interacting systems. In statistical mechanics, interactions between individual particles, spins, bodies, etc. lead to the macroscopic properties of materials. The dream of biological physics is to develop analogous theories to show how such interactions can lead to living matter. In the end, filling in the question marks might involve many intermediate steps composed of individuals models of the processes and components of a cell. Image inspired by Hyun Youk's work at http://youklab.org/research.html.

molecule which make up the cell. In this framing, the large number of degrees of freedom in a cell is only one aspect which makes such systems difficult to analyze. In a way the task the physicist faces in developing theoretical models of biology is much like the task of a surveyor who is sent out to explain the basic operations and economics of an entire city after he has spent his entire career managing the weekly allowances of children. In short, the task is both quantitatively different in scale and qualitatively different in the diversity of separable problem contexts.

In this difference one must admit that the large number of degrees of freedom present in a cell are correlative rather than causative of the complexity we observe. This is as opposed to statistical mechanics in which the $N \to \infty$ limit is a foundational assumption of the theory. However, the fact that this complexity correlates with the large number of degrees of freedom suggests that even if the modern concept of emergence is not the end-all framework for understanding the relationship between life and physics, it is at least a useful starting point. But of course, something "more" is at work here.

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