Indeterminacy and New Phenomena

These notes are part of a series concerning "Motifs in Physics" in which we highlight recurrent concepts, techniques, and ways of understanding in physics. In these notes we discuss and provide examples of how introducing indeterminacy into a system leads to phenomena which are not present in the original non-deterministic analog of the system.

Demons of Determinism

In 1814 [1], Laplace made an early contribution to the field of thought of experiments by conjuring up an intelligent entity who was capable of understanding physics. In fact this entity was so intelligent and so adept at comprehending the patterns of our physical world, that Laplace claimed that such a being would be able to predict, to arbitrary precision, everything that was happening in the universe then and in the future, assuming that the being was given all the initial positions and forces of particles in the universe.

Laplace's image of a masterful being (subsequently labeled a "demon" by other scientists) carefully and correctly working out the future dynamics of all physical systems was a woefully mechanistic vision of the universe, but it was one which was a product of its time. The clockwork universe envisioned by Newton allowed no room for imprecision in prediction. Such a universe was binary at least as far as physical results were concerned: By the laws of physics, a launched projectile would or would not be at a found at a certain height at a certain time. There was no in between, no room for probability or indeterminacy.

Now, the story goes that Laplace's demon was vanquished in the early 20th century by quantum physics, but in fact the ideal of a fully deterministic universe was relinquished a few decades after Laplace conceived of it. Key to Laplace's vision of the universe was the notion of reversibility, the belief that not only is the future uniquely determined by the past, but the past is also uniquely determined by the future. However, the mid-19th century development of statistical mechanics and thermodynamics introduced the concepts of entropy and irreversibility and thus, in contradiction to Laplacian reversibility, affirmed the idea that there are systems whose past dynamics cannot be determined by their current dynamics.

But beyond the basic fact that there are some systems in which precise Newtonian-like predictions of particle motions are not possible, such random systems often exhibit phenomena quite foreign to their deterministic antecedents. Indeed we often find that imbuing a system with disorder or randomness, where this randomness is precisely modeled in some well-defined physical formalism, leads to predictions of new physical phenomena. This fact is so pervasive throughout physics it deserves to be stated on its own.

> **Indeterminacy and New Phenomena:** Imbuing a system (or merely parts of that system) with a probabilistic nature, allows previously non-interacting parts of the system to interact. In other words, the indeterminacy in the system makes possible previously impossible phenomena.

In the following sections we will discuss a number of examples which exhibit indeterminacy and consequently have properties and predictions not present in the corresponding deterministic system. We will begin with a toy example to motivate the idea, and then turn to examples from physics. The most common physical examples consistent with this motif are found in quantum theory, but we will discuss two non-quantum examples to demonstrate the idea's generality.

Toy Model of Randomness

To develop a heuristic sense of the way randomness can lead to new interactions, we consider the following toy model. Say we have three people: Alan, Billy, and Carl who interact through talking. The general system is defined as follows:

1. Alan and Billy can talk directly to each other, and Billy and Carl can talk directly to each other, but Alan and Carl cannot talk directly to each other.
2. Alan's and Carl's positions are fixed. Billy can move, but his position is either precisely known or defined by a probability distribution.
3. What is communicated between two people is instantaneous and specific to the moment of communication.

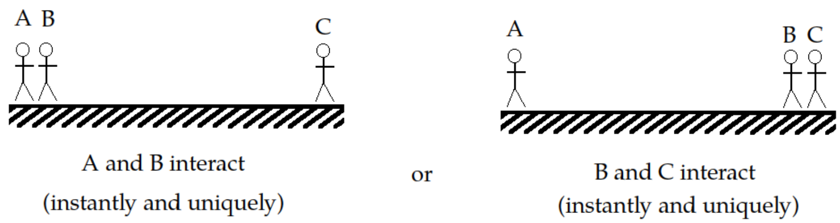


Figure 1: Interaction between A and B is not probabilistic.

In the "deterministic" case, we assume that Billy's position is precisely known. We can therefore postulate that Alan and Billy communicate if they are in the same position as do Carl and Billy if they are in the same position. This situation is depicted in Fig. 1.

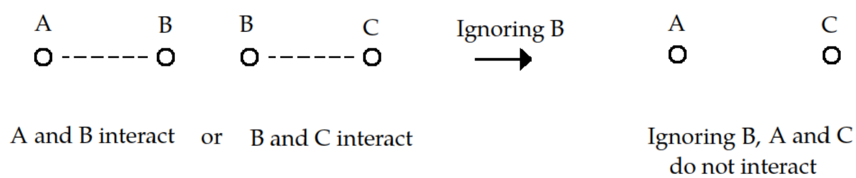


Figure 2: A and C have mutually exclusive interactions with B, so they have no effective interaction.

In such a scenario, Alan can talk to Billy, or Billy can talk to Carl, but because the conditions defining each of their interactions are mutually exclusive (given that Alan and Carl can't move), nothing Alan says to Billy will ever be communicated to Carl and vice versa. In essence, if Billy were some how operating invisibly in the background, Alan and Carl would not seem to communicate at all (See Fig. 2).

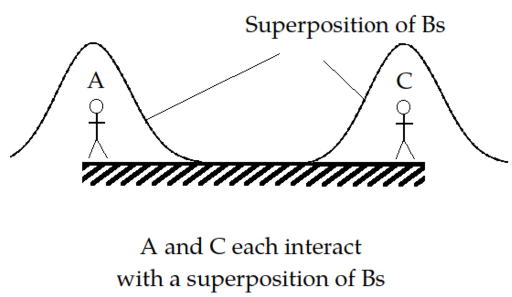


Figure 3: Interaction between A and B and between B and C is probabilistic.

In the "random" case, we assume we don't precisely know Billy's position. There is instead a probability distribution associated with Billy's position and hence a probability distribution associated with each of his possible interactions with Alan and Carl. This situation is depicted in Fig. 3.

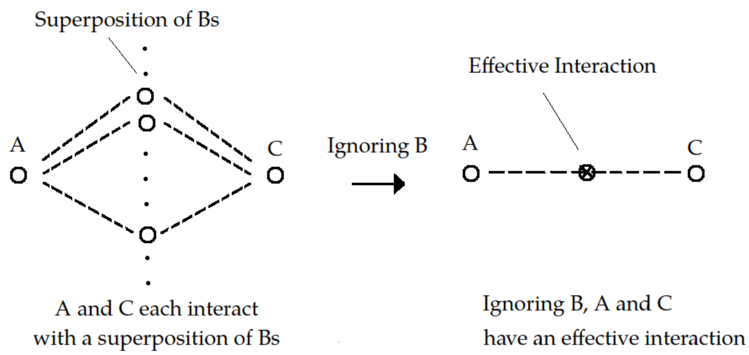


Figure 4: A and C each interact with a superposition of Bs, which leads to their effective interaction.

Over many different choices of Billy's position in this system, we would find a certain probability density of interaction between Alan and Billy and between Carl and Billy as shown in Fig. 3. In this situation, we could compute the probability that Alan interacts with Billy *and* the probability that, at the same time[1], Billy interacts with Carl for each choice of x_B. Then we could sum over different choices of x_B, weighted by the probability of effective interaction, to find the total probability of interaction between Alan and Carl. This procedure is represented schematically in Fig. 4. In the end we would find that if Billy were operating invisibly in the background, Alan and Carl would still be found to interact.

The most important aspect of this example is the introduction of a superposition of states to account for the probabilistic nature of the system. In the deterministic scenario, there is no superposition because states are known precisely. Thus mutually exclusive interactions between two parts of the system lead to the two parts being effectively separated. But by introducing randomness, we are forced to sum over various manifestations of the system weighted by each one's probability of occurrence. Such summations lead to effective interactions between parts of the system which would not otherwise interact.

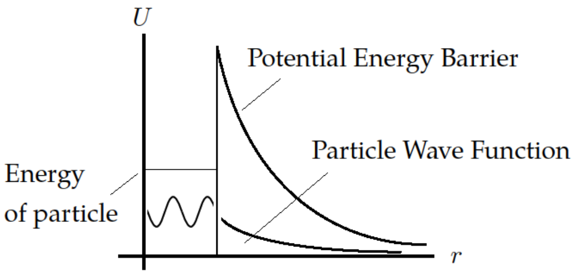
The Strange Quantum

Quantum phenomena are often so strange and non-intuitive that one famous physicist claimed that "nobody understands quantum mechanics" [**?**]. And yet contained within quantum mechanics is an idea which is generalizable beyond quantum mechanics itself: New phenomena result when we make our system probabilistic.The toy example above presented the basics of this idea, and in the list below we demonstrate it through four examples from quantum mechanics.

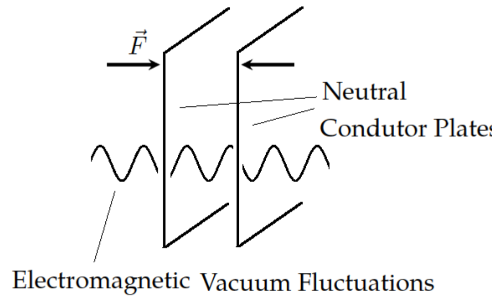
- **Quantum Tunneling:** From classical mechanics, we know that a particle in a potential energy $U(x)$ must have energy $E > U(x)$. Indeed such a particle could not exist in any region where the potential energy exceeds the energy. However, in quantum mechanics, a particle which classically does not have enough energy to cross a potential energy barrier, can do so (although with exponentially decaying probability). For example, alpha decay—the tunneling of an alpha particle through the nuclear potential—occurs by this process. Fig. 5a.

- **Casimir Effect:** From Coulomb's law, two electrically neutral and static objects should not experience an electromagnetic interaction. However, when we consider the zero-point energy of the electromagnetic field—a zero-point energy predicted by quantum electrodynamics and which is analogous to the harmonic oscillator ground state—we find that there is a nonzero force between the two plates. This phenomena (which has been experimentally verified) is known as the Casimir effect. Fig. 5b.

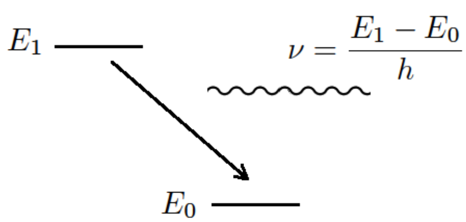
[1] For the sake of this example, we ignore the fact that such an interaction framework could violate causality.



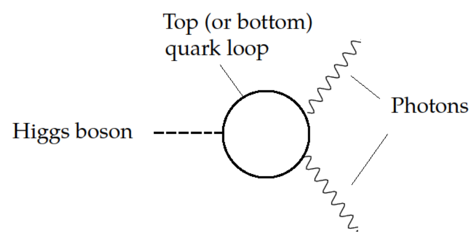
(a) Quantum Tunneling



(b) Casimir Effect



(c) Spontaneous Emission



(d) Higgs Boson Decay into Photons

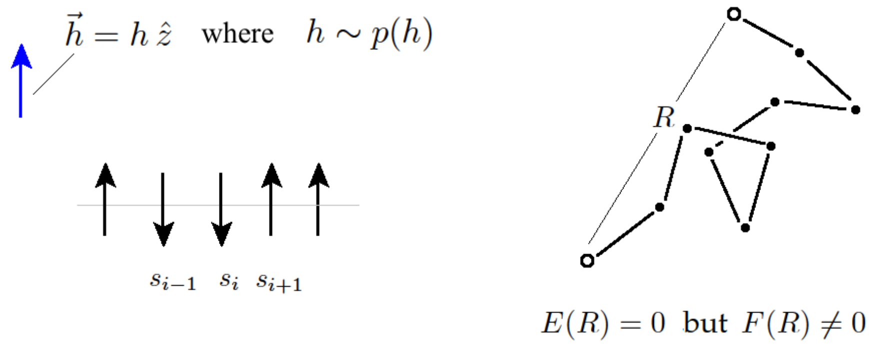
Figure 5: Quantum processes which do not occur in classical theory.

- **Spontaneous Emission:** The random decay of an atomic state to a lower orbital and the subsequent release of photons, should not occur if our electromagnetic field were purely classical: An unperturbed atom in an eigenstate of its Hamiltonian should remain in that state. However, the electromagnetic field is not purely classical. Rather space is filled with the zero-point energy of the quantum electromagnetic field—the same zero-point energy responsible for the Casimir effect—which continually perturb the atomic state. These perturbations can jostle the atom out of its original state producing a photon in the process. Fig. 5c.
- **2012 Discovery of Higgs:** The Higgs boson is an electrically neutral particle, and therefore, from a classical perspective, should not interact with photons. However, quantum mechanically, the Higgs can decay into intermediate quark states which then couple to photons thus producing an effective Higgs-photon interaction. This effective interaction exists at the foundation of the $H \to \gamma + \gamma$ decay which was one of the processes through which the Higgs was discovered in 2012. Such a process (shown in Fig. 5d) cannot occur classically.

Disorder and Thermal Fluctuations

Although quantum systems are perhaps the most natural physical example of how indeterminacy leads to new interactions, there are many non-quantum systems where randomness leads to novel system properties. We will discuss two physics examples here.

- **Spins and Disorder**
  The standard physics picture of a permanent magnet consists of a system of spins arranged in a lattice and interacting through a fixed coupling. Depending on the temperature and dimensionality of the system, the spins could be in a "ferromagnetic phase" where all or most spins are aligned along a



(a) Spin System

(b) Free Polymer Chain

Figure 6: Interactions which arise from disorder and thermal fluctuations. In Fig. 6a, non-interacting spins are all collectively subjected to the same random magnetic field and can then be represented as having an effective interaction. In Fig. 6b, a polymer chain in which each segment does not interact with any other has an energy $E(R)$ of 0 as a function of the end-to-end distance R. But the free energy $F(R)$ defines the thermal equilibrium state and is non-zero due to the entropy of the configuration.

specific direction or a "paramagnetic phase" where there is no direction of majority alignment without the presence of a magnetic field.

If, however, we say that the interactions have disorder, that is if the interactions between spins are drawn from a probability distribution, then a new phase of matter called the **spin glass phase** results. In the spin glass phase, small clusters of spins are aligned along a specific direction, but the average spin of the entire lattice is still zero.

Here's a simple theoretical example (unrelated to spin glasses) of how random interactions in spin systems can lead to new phenomena. Consider the thermodynamics of a collection of N non-interacting spins each coupled to the same magnetic field h (Fig. 6a). The partition function for such a system is

$$Z(h) = \sum_{\{s_i\}} \exp\left(\beta h \sum_{i=1}^{N} s_i\right). \tag{1}$$

For the system defined by Eq.(1), we see that different spins in the system do not interact because the argument of the exponential (representing the energy of the system) only contains terms linear in s_i. However, by making h a random variable[2], we will find an effective interaction between different spins.

Promoting h to a random variable requires that we associate it with a probability density. As a working example, we assign to h the density

$$p(h) = \frac{e^{-h^2/2\sigma_h^2}}{\sqrt{2\pi\sigma_h^2}}, \tag{2}$$

where σ_h^2 is the (possibly temperature dependent) variance in the field. Averaging the partition function Eq.(1) over the possible values of h given in Eq.(2), we find the resulting effective partition function between spins is

$$Z_{\text{eff}} = \int_{-\infty}^{\infty} \frac{dh}{\sqrt{2\pi\sigma_h^2}}\, e^{-h^2/2\sigma_h^2} Z(h)$$

[2]Making h a random variable in this way is akin to saying it fluctuates on the same time scale that the individual spins s_i fluctuate.

$$= \sum_{\{s_i\}} \int_{-\infty}^{\infty} \frac{dh}{\sqrt{2\pi\sigma_h^2}} e^{-h^2/2\sigma_h^2} \exp\left(\beta h \sum_{i=1}^{N} s_i\right)$$

$$= \sum_{\{s_i\}} \exp\left(\frac{\beta^2 \sigma_h}{2} \sum_{i,j} s_i s_j\right). \tag{3}$$

We note that Eq.(3) has an exponential in which the argument (again representing the energy) has terms multiplying two different spins. Thus, we thus see that the random magnetic field has led to effective interactions between spins which were independent when the field was not random. We note as well that these effective interactions go away as the uncertainty σ_h in the field goes to zero.

- **Polymer Models**
  The statistical mechanics of polymer systems is formally studied through what is known as **ideal chain models**. In the simplest form of these models, many individual straight line segments are joined together to form a chain in free space. There is no energy of interaction between each segment, so the total energy $E(R)$, where R is the end-to-end distance of the chain, of such a system is zero. But when we consider the thermodynamics of this system (by coupling it to a constant temperature heat bath, for example), the chain obtains a non-zero free energy $F(R)$ arising from the thermodynamic entropy of the system (Fig. 6b). In thermodynamics, the free energy, rather than just the energy, determines the thermal properties of a system, and in this case the free energy leads to an attractive force between the two ends of the chain. This force arises specifially from the thermal randomness of the system: As the temperature of the ideal chain goes to zero, so does the thermodynamic entropy, and thus so does the attractive force between the two ends.

Different types of Randomness

From the above examples, we have seen how systems with randomness or uncertainty can exhibit phenomena their non-random analogs do not. However, the reader may reasonably find issue with our grouping of the disparate phenomena just discussed under the common title of ”indeterministic systems”. Specifically, the randomness in thermal systems is of a different type of randomness from that which defines spin glasses, and both thermal randomness and glassy randomness are quite different from quantum randomness. Indeed, glassy randomness is a randomness in the interaction parameters which define a material and is, as such, distinct from the randomness of a physical process. Moreover, while the randomness of thermodynamics arises from an inherently non-random classical theory, the randomness of quantum mechanics is (as far as we know) completely fundamental.

Still, the intention of this discussion was not to conflate many fundamentally different sources of indeterminacy, but to show that models which exhibit indeterminacy bear qualitatively similar properties in spite of their distinct underlying physical mechanisms. The main property we find is that such random systems lead to couplings and interactions between parts of the system which would otherwise not interact.

References

[1] P. Simon, M. de Laplace, F. W. Truscott, and F. L. Emory, *A philosophical essay on probabilities*, vol. 166. Dover Publications, 1951.