Physics 95 (F17) - Models, Physics, and Life Science – Solution

due Sunday October 1, at 6pm

1. **Designing a Lecture Hall**
   The purpose of this task is to get you to work through a modeling problem using relatively basic mathematics. The skills you will employ here are the skills people are often referring to when they say "thinking like a physicist".

   Unless you have been lucky, you have had a large class in a poorly designed lecture hall.

   (b) One criterion is legibility of material written on the boards. Construct a model of legibility as a function of the distance your seat is from the board and the angle at which you look at the board. What will the curves of constant legibility look like on a floor plan? How could one test this prediction? What does this suggest about how the back of the hall should be shaped?

   (Length: No more than two pages)

We begin with the figure shown below.

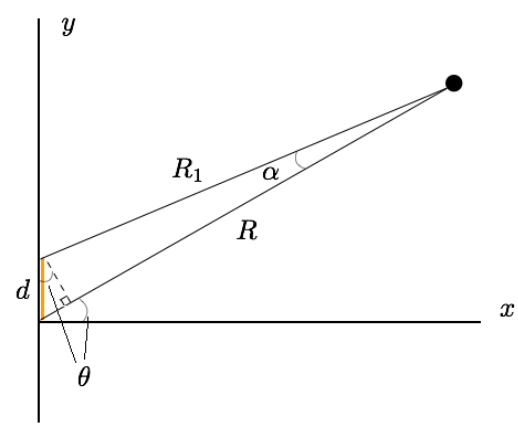

Figure 1: Top view of classroom. The front of the blackboard coincides with the vertical y axis. The orange strip defines text of diameter d written on the board. The black filled circle is a student. In polar coordinates, the students is at the position (r, θ). The variable r is also the distance between the student and the left side of the text. The variable R_1 is the distance between the student and the right side of the text. The angle α subtends the arc of length d when viewed from the student. α defines the visual resolution of text on the board and thus serves as a proxy for legibility. For the moment, we will ignore variations in the seating height of the students.

According to the figure, the variable α defines the angular resolution of text on the board as seen from a student in the lecture hall. We will let α stand in for our definition of legibility:

$$\text{Legibility} \equiv \alpha \tag{1}$$

We use this definition because it is reasonable to expect that all students who experience the same angular resolution can read the associated text equally well.

Now to determine the curves of constant legibility, we need to determine the curves in (r, θ) space associated with a constant α.

We begin by using the law of cosines. We have

$$R_1^2 + r^2 - 2R_1 r\cos(\alpha) = d^2. \tag{2}$$

Solving for $\cos(\alpha)$, which is related to our proxy for legibility, we have

$$\cos(\alpha) = \frac{R_1^2 + r^2 - d^2}{2R_1 r}. \tag{3}$$

For typical classrooms, we can imagine the text on the board to be quite small relative to the student's distance from the text. This, in turn, means the angular resolution satisfies $\alpha \ll 1$. This allows us to make the approximation

$$\cos(\alpha) = 1 - \frac{\alpha^2}{2} + \mathcal{O}(\alpha^4). \tag{4}$$

Using the figure, we can establish the relation $r - d\sin\theta = R_1 \cos\alpha$, and again given $\alpha \ll 1$, we find

$$R_1 = r - d\sin(\theta) + \mathcal{O}(\alpha^2). \tag{5}$$

With Eq.(4) and Eq.(5), we can show that Eq.(3) reduces to

$$\alpha = \frac{d}{r}\cos(\theta) + \cdots, \tag{6}$$

where $\cdots$ are terms arising from higher order terms in Eq.(4) and Eq.(5). We can neglect these terms up to a desired order of approximation. For constant legibility we set $\alpha = \alpha_0$. Therefore, the equation defining constant legibility (as viewed from the top of the classroom) is

$$r(\theta) = \frac{d}{\alpha_0}\cos(\theta). \tag{7}$$

Using $r^2 = x^2 + y^2$ and $\cos\theta = x/r$, we find the cartesian representation of this equation is

$$\left(x - \frac{d}{2\alpha_0}\right)^2 + y^2 = \frac{d^2}{4\alpha_0^2}, \tag{8}$$

which is the equation of a circle of radius $d/2\alpha_0$. Thus, to have constant legibility (at the same height) students should be seated along the arc length of a circle.

Considering Seating Height

If we were to take a side view of the classroom rather than a top view, we would obtain the same result, i.e., that Eq.(8), this time from a side view, defines the curves of constant legibility . But gravity of course limits the possibility of using large arc-lengths as the ground plans for the leveled seating of a lecture hall. However, this result suggests that the constant-elevation inclined plane method of creating leveled seating is not the best one for optimizing student legibility.

Testing

We could test Eq.(8) by mapping the coordinate system in Fig. 1 onto a large lecture hall and marking the outlines of a circle defined by Eq.(8). (We would need to define α_0 beforehand.) Then with text written on the board and a camera chosen at a fixed zoom, we take pictures of the text from various points in the classroom both on and off the arc-lengths defined by Eq.(8). Comparisons of the resulting text sizes in the developed film (e.g., all pictures taken on the arc-length should produce the same size text) would then determine the validity of Eq.(8).