

Problem Solving: Inside and Outside the Classroom

An education in physics is often touted as doubling as an education in problem solving, but the types of problems physics students are trained to solve do not well simulate—even in their general outline if not in the specific situations they consider—the problems students may encounter in less artificial contexts. The main source of this mismatch is that assigned physics problems often require students to engage with only a part of the problem-solving process: the part where one seeks a solution to a well-defined question. Formulating a well-defined question and recognizing when such a question should be formulated are skills just as important (if not more so) than the standard skills which are typically considered definitive of problem solving, and if the student wants to become an effective problem solver he or she should create opportunities to practice them.

1 Physics and Problem Solving

Because understanding in physics is often directed toward a goal beyond a mere display of knowledge, the best way to learn physics is to practice it in the way one hopes to someday use it: through solving problems.

Consistent with this idea, this class (and mostly every other physics class) adopts a problem-based approach to physics education where the student is tasked with independently and carefully using her knowledge, gained from lectures and textbooks, to answer questions she hasn't encountered before. The process of struggling through such questions instills a solid working knowledge of the principles of physics and therefore allows the student to walk away with a much better understanding of physics than reading and rereading a relevant text could ever provide.

Moreover, there are supposed fringe benefits of such an education. Physics, as it is often abstract and mathematical, is often touted as an excellent arena in which to practice the general methods of problem solving. Perusing through many physics department or physics related websites [1] you can see arguments for a physics major based not only on the deeper understanding of the world the student could obtain but also on how the student inevitably becomes a better problem solver. Such a notion is largely the basis of the argument for why a physics major is valuable to a career unrelated to physics or even science.

But it is not completely true. Going through a physics course will not in general make you a better problem solver because the problems you learn how to solve do not well model the messier problems you encounter outside the classroom.

Take a problem from David Morin's excellent text *Introduction to Classical Mechanics* [2]. This problem shares many similarities in style to problems in this course and virtually most courses in physics.

A billiard ball collides elastically with an identical stationary one. By looking at the collision in the CM [Center-of-Mass] frame, show that the angle between the resulting trajectories in the lab frame is 90 degrees. (Morin 2008, pg 191)

This type of problem is typically characterized as “well-defined” because for each decision the student may have had to make to solve the problem, the problem statement already suggests the outcome of the decision. First, there is the key word “elastic” which means we should assume that this process conserves

kinetic energy in addition to conserving momentum. Second, there is the suggestion that we study the collision in the center-of-mass frame; studying the collision in this frame is not necessary, but doing so certainly simplifies the algebra needed to obtain a solution. Finally, the problem states that we should calculate the angle between the two collision trajectories and confirm that the resulting angle is 90 degrees in the lab frame.

Of course, even given the elaborate problem statement, the problem is still unsolved, and the student would still need to work through a few careful calculations to obtain a solution. But, largely, the ambiguous aspects of the question have been made concrete, and the student mostly needs to work algorithmically to find an answer.

The main artifice of this problem and those like it extends in two directions.

1. The problem gives an explicit framework (replete with assumptions and/or approximations) which states clearly what question you're answering.
2. The problem suggests what techniques should be used to answer the question.

In real problems, you generally don't know what questions you should be asking or the best ways to answer them, and you consequently have to do quite a bit of exploring to resolve both.

To put it summarily: **this problem and most of the problems you'll solve in this class¹ differ in scope and precision of definition from the problems you would likely encounter in any other less contrived academic or non-academic context. Because these problems only require you to practice a part of the problem-solving process—the part where you answer a well-defined question—the problem-solving practice you obtain through them does not cultivate general problem-solving ability.**

What are the other steps of the problem-solving process? They consist of, first, recognizing that a problem exists and, second, transforming that recognition into a concrete problem framework (e.g., a well-defined question) which could yield an acceptable solution.

In order to obtain a better sense of the true nature of this process, we give an example of a scientist solving a problem. Of course scientists are not the only ones who solve problems outside of classrooms and not all students who study physics want to become scientists, but if the problem-solving education in physics has any wider relevance outside of physics it is supposedly because it well simulates the approach a trained scientist takes when encountering a problem.

We'll use the (arguably unrepresentative) example of Schrödinger developing and eventually solving the problem of the quantum description of the hydrogen atom. From this example we will extrapolate and discuss the true steps of the problem-solving process.

2 The Example of Schrödinger

We will outline Schrödinger's path to the Schrödinger equation to illustrate how in real problem solving an investigator moves from a state of relatively great ambiguity to a state of great concreteness, at which latter point he may solve problems of the type typically practiced in a physics class. But, before we discuss Schrödinger, we summarize part of the historical context which framed his work.

¹ save, perhaps, for a problem you choose for your final project

In 1900 Max Planck, in an attempt to explain the spectrum of frequencies produced by a perfectly emitting heated body (i.e., a “black body”), posited that the energy E of light emitted at each frequency ν was quantized in amounts given by

$$E = h\nu, \quad (1)$$

where h is Planck’s constant. In his recollections of this work, Planck stated that he had little other motivation to introduce this idea other than his frustration to establish the viability of alternatives. And yet, from this postulate, he was able to derive the famous blackbody radiation formula for the density spectrum of light emitted from a black body. However, because his postulate of energy quantization was introduced more so out of pressing need to find semblance with experimental results than out of any deep physical insight, Planck was reluctant to interpret physically the implications of his formalism.

Interpretation and understanding came later in 1905 through Einstein’s work on the photoelectric effect. Attempting to explain why electrons are ejected from a metal when light shines on it, Einstein fully committed to Planck’s discrete energy postulate Eq.(1) and further postulated that the energy of light came in discrete quantities because in the quantum realm, light (termed “photons”) had particle-like properties.

Nearly, twenty years later, a Ph. D student named Louis de Broglie published his dissertation where he argued the converse of Einstein’s photoelectric effect postulate. Namely, taking Einstein’s work on photons and the newly developed theory of special relativity [3], de Broglie argued that similar to the way electromagnetic waves can have particle-like properties, particles can have wave-like properties. Specifically, he established that a quantum particle of a certain momentum p carried with it a wave with wavelength λ given by

$$\lambda = h/p \quad (2)$$

This was the state of affairs when, in November of 1925, Peter Debye urged Schrödinger to review de Broglie’s recently published thesis in a biweekly research seminar.

To prepare for the review, Schrödinger worked through Debye’s thesis and sent letters to Einstein and other physicists expressing his awe of the work and his desire to obtain a better sense of what it really meant to consider a quantum particle as a wave. In a letter to Alfred Landé, Schrödinger writes [4]

I have been intensely concerned these days with Louis de Broglie’s ingenious theory. It is extraordinarily exciting, but still has some very grave difficulties. I have tried in vain to make for myself a picture of the phase wave of the electron in the Kepler orbit. Closely neighboring Kepler ellipses are considered as rays. This, however, gives horrible ‘caustics’ or the like for the wave fronts.

In this correspondence, we see Schrödinger recognizing the conceptual potency of de Broglie’s results while simultaneously struggling with an inability to understand and hence fully utilize that potency. It seems Schrödinger’s main road block was he did not know the correct question to ask.

The right question came during the seminar where Schrödinger finally presented de Broglie’s wave-particle thesis. In the seminar, Debye mentioned that de Broglie’s wave-particle duality ideas were interesting but woefully imprecise. In his opinion, any system that truly had wave-like properties should also have a wave equation.

Thus Schrödinger’s task was set: Find a wave equation for subatomic particles like the electron. Weeks

later he used the energy-momentum relation from special relativity to obtain the time-independent Klein-Gordon equation

$$(E - V)^2\psi(\mathbf{x}) = -\hbar^2\nabla^2\psi(\mathbf{x}) + m^2\psi(\mathbf{x}), \quad (3)$$

where E is the energy, V is the potential energy, m is the mass, and $\psi(\mathbf{x})$ was a function he had not yet learned how to interpret. After he obtained the Klein-Gordon equation, Schrödinger made his problem well-defined by setting $V = -e^2/r$ for the potential energy of the hydrogen atom and computing E , the associated energies. But he found that this associated energy spectrum did not match the observed spectrum, and so he abandoned this formalism and began looking for another.

Over his Christmas vacation, Schrödinger reformulated his approach and obtained what we now call the time-independent Schrödinger equation. He applied the equation to the hydrogen atom and found that it produced the desired spectrum:

$$E\psi(\mathbf{x}) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}) - \frac{e^2}{r}\psi(\mathbf{x}) \quad \text{gave} \quad E_n = -\frac{me^4}{2\hbar^2} \frac{1}{n^2} \text{ for } n = 1, 2, \dots \quad (4)$$

This result agreed with the spectra of hydrogen discovered in the late 19th century. With this affirmation, Schrödinger then had the confidence to extend his equation in other directions and further build toward what we now consider modern quantum mechanics.

3 Education and the Problem-Solving Process

From this historical example, we glean that the problem-solving process begins well before the investigator has any concrete sense of what exactly “the problem” is. One typically begins by recognizing a gap in one’s current understanding of a system or some need which is unfulfilled. In other words, one is in a state of confusion, inconsistency, or relative need, and there is a sense (ranging from explicitly apparent to vaguely felt) that something is missing in one’s understanding the world.

From this state, one then tries to make this missing element concrete by articulating it within a more explicit framework. The first steps at articulation may bear the marks of the initial haziness of the one’s ideas about the problem, but by successively constructing and interrogating frameworks, one can gradually move closer to a framework that is concrete and answerable.

This concrete framework is what we often label as “the problem” itself. After it is developed, one can then search for a solution using all the standard techniques [5] (e.g., “guess and check”, “work backwards”, “draw a figure”, etc.) which are considered standard approaches to solving a well-defined problem. If the solution one finds is inadequate, one returns to the problem formulation stage and begins again.

In Schrödinger’s experiences we see the main hallmarks of this process. There was

- First, the recognition of a gap in knowledge: Schrödinger’s tentative search for a more concrete picture of de Broglie’s ideas and Debye’s suggestion that a wave equation is necessary to precisely describe wave-particle duality
- An imposition of a problem framework to address that gap: Schrödinger’s development of the Klein-Gordon equation for the hydrogen atom

- A solution of that framework: Schrödinger’s calculation of the Klein-Gordon hydrogen atom spectrum which incorrectly predicted the known results
- And when that solution was found to be inadequate, a repetition of the process from the problem framework stage: Schrödinger’s development of a non-relativistic version of the wave equation and his later application to the hydrogen atom to obtain the correct spectrum

This process, along with labels for the parts taught in school, is depicted below in Fig. 1

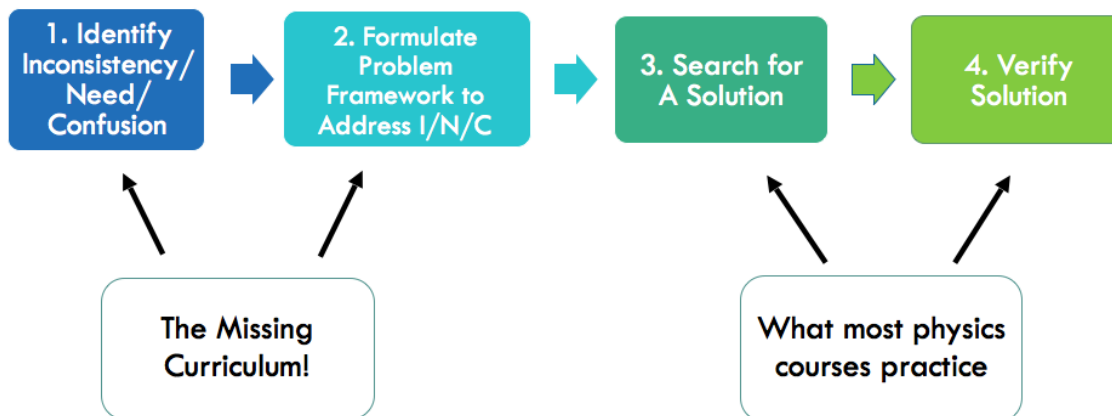


Figure 1: The steps of the problem-solving process: Most physics courses do not teach students how to identify gaps in understanding or how to use these gaps to create well-defined problems.

It is necessary to mention, as in Schrödinger’s case, one usually doesn’t proceed linearly through these steps but cycles through them as appropriate according to how one’s understanding of the original knowledge gap evolves.

As depicted in the figure, most physics courses (this course included) emphasize practicing the third and fourth step of this process: finding and checking solutions to well-defined problem frameworks. And yet if you were ever to use the knowledge you’ve built in these physics classes to answer a new question about the world, you would inevitably have to proceed through all the steps of the process.

There are philosophical and practical reasons for why physics courses focus on practicing the final steps, but mostly I just want you to be cognizant of this focus. And these notes were meant to be explicative rather than prescriptive so I’m not going to advocate specific ways to practice other steps in the sequence².

But in a way this class already will require you to practice these other steps. For the final research paper you will have to choose a topic (i.e., effectively identify a gap in *your* knowledge) and explicate the topic in a way which is understandable to your peers (i.e., effectively develop and solve a problem framework). Still it is worth recognizing that most physics courses (especially ones which have little student driven work) suffer from the deficiency cited here, so to practice the full spectrum problem-solving skills you will often have to look outside the physics classroom.

²But here are two books which are useful in this direction: *What If?: Serious Scientific Answers to Absurd Hypothetical Questions* by Randall Munroe; *Thinking Like a Physicist* by N. Thompson;

References

- [1] A. P. Society, "Why study physics," 2016.
- [2] D. Morin, *Introduction to classical mechanics: with problems and solutions*. Cambridge University Press, 2008.
- [3] L. De Broglie, L. de Broglie, L. de Broglie, and F. Physicist, *Recherches sur la théorie des quanta*. Masson Paris, 1963.
- [4] W. J. Moore and W. J. Moore, *A life of Erwin Schrödinger*. Cambridge University Press, 1994.
- [5] G. Polya, *How to solve it: A new aspect of mathematical method*. Princeton university press, 2014.