

NYT Digits - Algorithms and Mathematics

JFR Journal Club

Mobolaji Williams, July 25 2023

NYT Digits - Rules of the Game

Use any combination of numbers to reach the target:

52

1 2 3

4 5 25

↺ + - × ÷

Submit

1. You are given six numbers and a target number.
2. The goal is to reproduce the target number from any combination from the set of six numbers using the operations addition, subtraction, division, and multiplication.
3. In combining numbers, divisions cannot have remainders and subtractions cannot result in negative numbers.

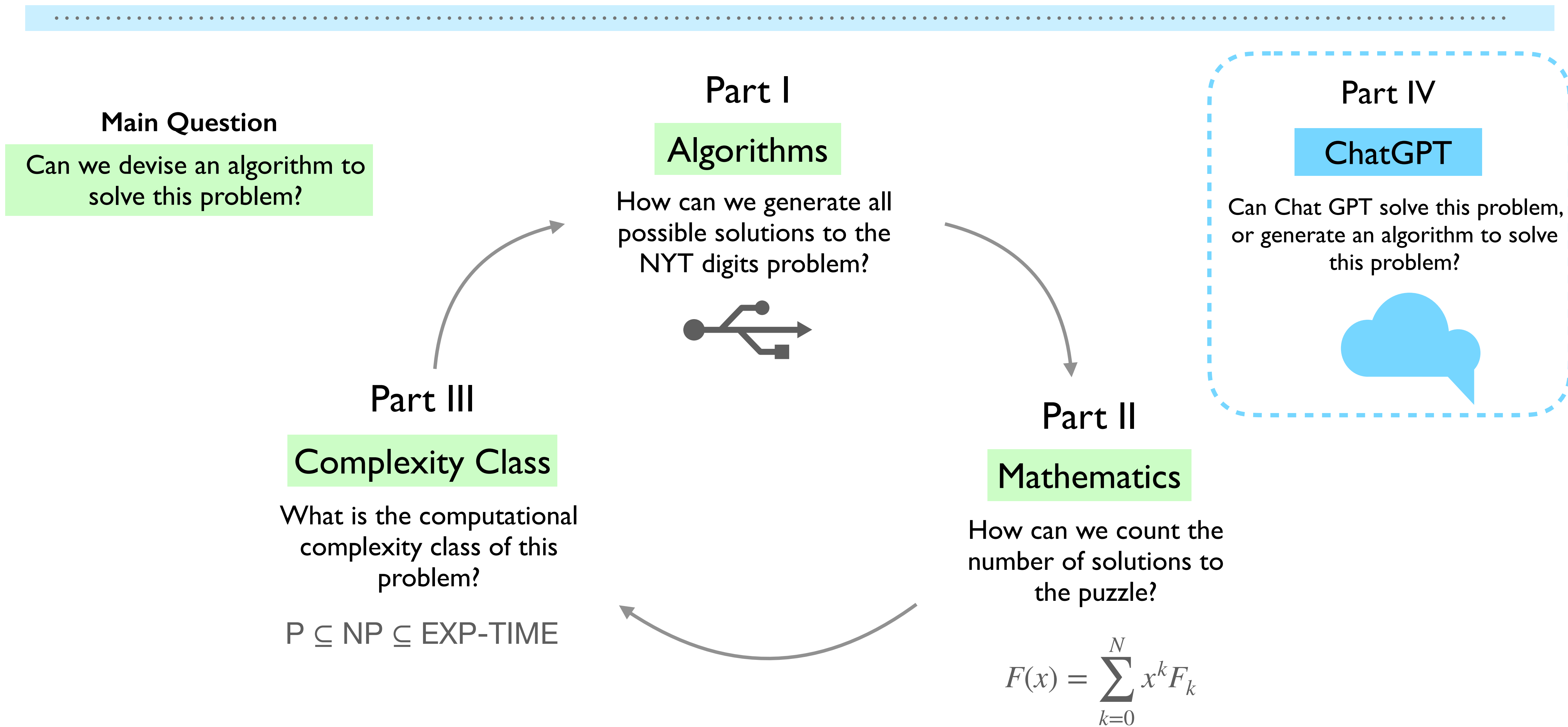
Main Question

Can we devise an algorithm to solve this problem?

Supplementary Questions

And what can the algorithm tell us about the nature of the solutions to this problem and this problem's computational complexity?

Presentation Outline



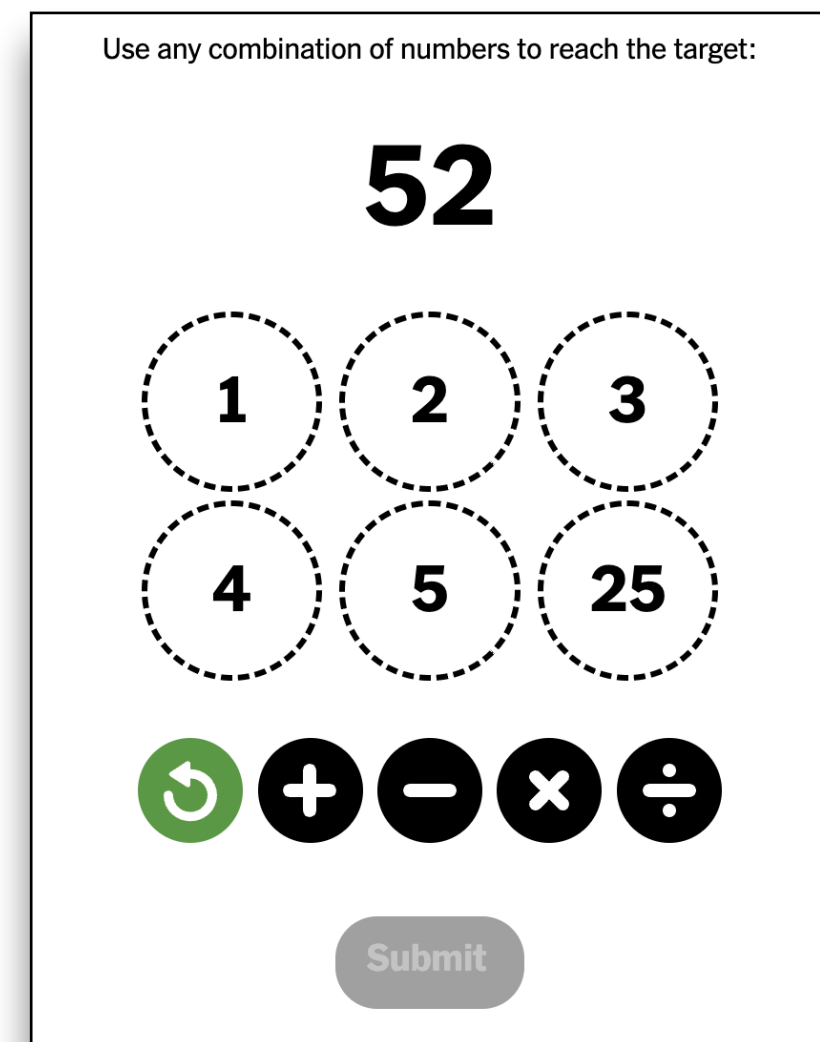
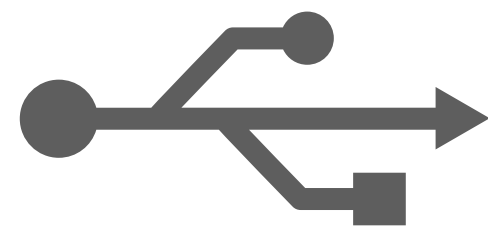
Part I: Algorithms

Part I: Incremental Programming

Part I

Algorithms

How can we generate all possible solutions to the NYT digits problem?



Refined Question: How can we find all valid computational combinations of **six numbers** that yield a target number?

We will use the principle of **incremental development**

“The goal of **incremental development** is to avoid long debugging sessions by adding and testing only a small amount of code at a time.”
- *How to Think Like a Computer Scientist*



Rather than solving the full “six number” puzzle, we try to solve the “**two-number**” puzzle, “**three-number**” puzzle, “**four number**” puzzle and so on.

Incremental Question: How can we find all valid computational combinations of **two numbers** that yield a target number?

Part I: Two-Number Combinations

Incremental Question: How can we find all valid computational combinations of **two numbers** that yield a target number?



If we already have the combinations, we can **easily** find the ones that match a given number. The **difficult** part is generating these combinations

Refined Incremental Question #1: How can we generate all valid computational combinations of **two numbers**?

Example:

We have the numbers 1 and 2

Possible Combinations:

$2 + 1$, $2 - 1$, 2×1 , $2 / 1$

But we want to keep track of both the “names” of the combinations and their “results” and so we want a function that does both tasks

Example:

```
Python   
1 gen_two_combos({'1': 1, '2': 2})  
2 >>> {'(2+1)': 3, '(2*1)': 2, '(2/1)': 2, '(2-1)': 1}
```

Part I: Two-Number Combinations

Refined Incremental Question #1: How can we generate all valid computational combinations of **two numbers**?

Example:

We have the numbers 1 and 2

Possible Combinations:

$2 + 1$, $2 - 1$, 2×1 , $2/1$

We want to keep track of both the “**names**” of the combinations and their “**results**” and so we want a function that does both tasks.

- Also want to **prevent** “bad” operations from leading to valid values

```
Python   
1 def gen_two_combos(value_dict):  
2  
3     # sort dictionary by value to ensure subtractions are not negative  
4     # and divisions are greater than 1  
5     sorted_vals = dict(sorted(value_dict.items(), key=lambda item: item[1], reverse=True))  
6  
7     # access the list of string names and the list of associated integers  
8     name_list, num_list = list(sorted_vals.keys()), list(sorted_vals.values())  
9  
10    # for writing simplicity  
11    elem1, elem2 = num_list  
12    name1, name2 = name_list  
13  
14    # generating all combinations  
15    # internal if conditional yields a large number if the division  
16    # and subtraction constraints are violated  
17    combo_dict = {'('+name1+'+'+name2+')':elem1 + elem2,  
18                  '('+name1+'*'+name2+')':elem1 * elem2,  
19                  '('+name1+'/'+'+name2+')': int(elem1/elem2) if elem1%elem2==0 else 1/3001,  
20                  '('+name1+'-'+'+name2+')': elem1- elem2 if elem1- elem2>0 else 1/3001 }  
21  
22    return combo_dict
```

names

results

prevention

```
Python   
1 gen_two_combos({'1': 1, '2': 2})  
2 >>> {'(2+1)': 3, '(2*1)': 2, '(2/1)': 2, '(2-1)': 1}
```

Part I: Three-Number Combinations

...solved the two number problem...

...next we have...

A Worked Example

Say we have three numbers 1, 2, and 3. What are the various ways we can combine these numbers using our four operations?

- If we select two numbers 1 and 2 and multiply them, then we would have the number 2 in addition to the original unselected 3.
- We would now have two numbers. We can then combine this new 2 and the old 3 in all the ways that two numbers can be combined.
- We know there are four ways to combine two numbers, so to find the various ways to combine three numbers, we can first ask how can we form two numbers from three numbers.

Refined Incremental Question #2: How can we generate all valid computational combinations of **three numbers**?

We can use the **two number** solutions for the **three number** solutions.

Answer: The various ways to combine **three numbers** consists of...

$$= \binom{3}{2} \text{ (4 ways)} \otimes \text{ the ways to combine those two selected numbers} \otimes \text{ (4 ways) the ways to combine the result of the two-number combinations with the third unselected number}$$

Binomial coefficient*

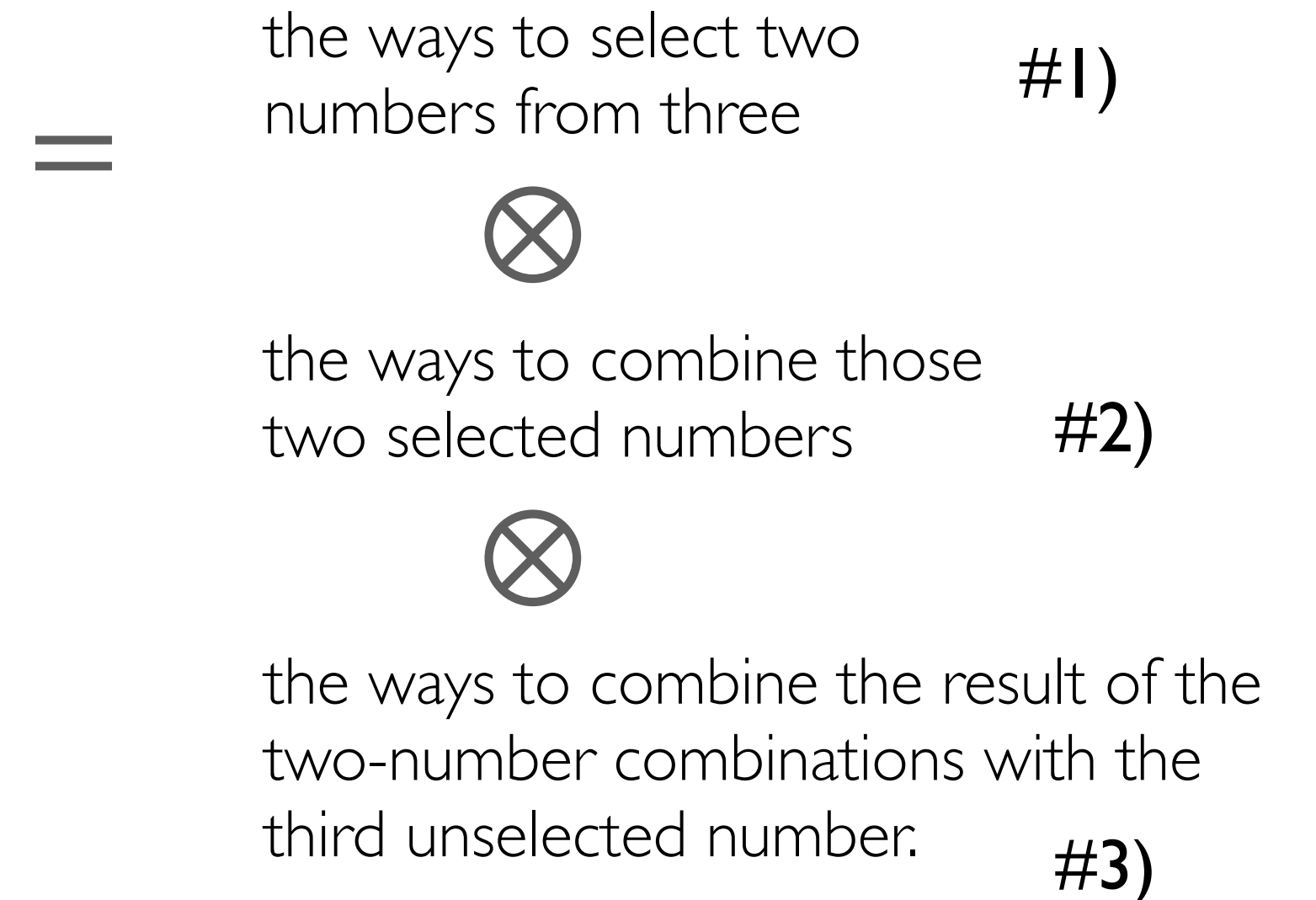
$$\binom{n}{k} \equiv \frac{n!}{k!(n-k)!}$$

Part I: Three-Number Combinations

Refined Incremental Question #2: How can we generate all valid computational combinations of **three numbers**?

We can use the two number solutions for the three number solutions.

Answer: The various ways to combine **three numbers** consists of...



```
1 def gen_three_combos(start_dict):
2
3     # empty dictionary to house combo outputs
4     output_dict = dict()
5
6     for key, value in start_dict.items():
7         # define copy of three number combination
8         copy_dict = start_dict.copy() #1)
9         del copy_dict[key] # eliminate one number from the three
10
11        # for all combinations of the two remaining numbers #2)
12        for name, elem in gen_two_combos(copy_dict).items():
13            # create combinations of the two-number result and the unselected number
14            new_dict = {name: elem, key:value }
15            result_dict = gen_two_combos(new_dict) #3)
16            output_dict.update(result_dict)
17
18    return output_dict
```

Example:

```
In [6]: gen_three_combos({'1': 1, '2': 2, '3': 3})
```

```
Out[6]: {'((3+2)+1)': 6,
          '((3+2)*1)': 5,
          '((3+2)/1)': 5,
          '((3+2)-1)': 4,
          '((3*2)+1)': 7,
          '((3*2)*1)': 6,
          '((3*2)/1)': 6,
          '((3*2)-1)': 5,
          '(1+(3/2))': 1.000333222259247,
          '(1*(3/2))': 0.0003332222592469177,
          '(1/(3/2))': 0.0003332222592469177,
          '(1-(3/2))': 0.9996667777407531,
          '((3-2)+1)': 2,
```

Part I: Four-Number Combinations

Refined Incremental Question #3: How can we generate all valid computational combinations of **four numbers**?

We use the same logic as that for the **three number** and **two number** combinations...

#1. Choose subsets of numbers

#2. Build up the solution up from the lower-order solutions

This procedure is reminiscent of something that is common in combination and programming problems...

Recursive programming: Build up a solution by referring to solutions to sub-problems

This helps us formulate the general solution

```
Python 
1  from itertools import combinations
2
3  def gen_four_combos(start_dict):
4
5      output_dict = dict()
6
7      # split four numbers into three numbers and one number
8      for key, value in start_dict.items():
9          copy_dict = start_dict.copy() #1)
10         del copy_dict[key]
11
12         for name, elem in gen_three_combos(copy_dict).items():
13             new_dict = {name: elem, key:value } #2)
14             result_dict = gen_two_combos(new_dict)
15             output_dict.update(result_dict)
16
17     # split four numbers into two numbers and two numbers
18     for elem in list(combinations(start_dict.keys(), 2)):
19         copy_dict = start_dict.copy()
20         del copy_dict[elem[0]] #1)
21         del copy_dict[elem[1]]
22         comp_copy_dict = {elem[0]:start_dict[elem[0]], elem[1]:start_dict[elem[1]]}
23
24         for key1, value1 in gen_two_combos(copy_dict).items():
25             for key2, value2 in gen_two_combos(comp_copy_dict).items():
26                 new_dict = {key1: value1, key2:value2 }
27                 result_dict = gen_two_combos(new_dict) #2)
28                 output_dict.update(result_dict)
29
30     return output_dict
```

Part I: General Combinations

Refined Incremental Question #4: How can we find all valid computational combinations of **any list of numbers***?

```
Python 
```

```
1 from itertools import combinations
2 from math import comb
3
4 # generates all manipulation combinations of a dictionary of elements
5 def generator_combos(start_dict: dict):
6
7     # defining dict_len to determine pathing
8     dict_len = len(start_dict)
9
10    # defining output dictionary
11    output_dict = dict()
12
13    count = 0
14
15    # output the dictionary if there is just one element
16    if dict_len==1:
17        output_dict = start_dict
18
19    # output all ways to add, subtract, multiply, or divide two numbers
20    elif dict_len==2:
21        # sort to ensure division is >= 1 and subtraction is >= 0
22        sorted_vals = dict(sorted(start_dict.items(), key=lambda item: item[1], reverse=True))
23
24        # list of names and numbers
25        name_list, num_list = list(sorted_vals.keys()), list(sorted_vals.values())
26
27        # getting elements and there names
28        elem1, elem2 = num_list
29        name1, name2 = name_list
30
31        # set of all possible operations with keys for operation name
32        # Note: digits game doesn't allow fractions or negative numbers
33        # so we set the result to a large
34        output_dict = {'('+name1+'+'+name2+')':elem1 + elem2,
35                       '('+name1+'*'+name2+')':elem1 * elem2,
36                       '('+name1+'/'+name2+')': int(elem1/elem2) if elem1%elem2==0 else 1/3001,
37                       '('+name1+'-'+name2+')': elem1- elem2 if elem1- elem2>0 else 1/3001 }
38
```

*We are generalizing the problem to any list of numbers

```
38
39 # for dict_len >=3, recursively build sub solutions
40 else:
41
42     # only go up to half the number of elements in dict
43     # to prevent double counting (e.g., n choose k = n choose n-k)
44     for ix in range(1, dict_len//2+1):
45
46         # number of ways to select 'ix' elements from 'dict_len' total
47         num_comb = comb(dict_len, ix)
48
49         # limiting number of combinations considered
50         # when there is an even split; this prevents double counting
51         if ix == dict_len/2:
52             lim = num_comb//2
53         else:
54             lim = num_comb
55
56         # generating combinations of elements
57         # '[:lim]' prevents double consideration of combos
58         for elem in list(combinations(start_dict.keys(), ix))[:lim]:
59             copy_dict = start_dict.copy()
60             comp_copy_dict = dict() # complement of copy_dict
61             for j in range(ix):
62                 del copy_dict[elem[j]]
63                 comp_copy_dict[elem[j]] = start_dict[elem[j]]
64
65             # generating set of possible operations for elements
66             for key1, value1 in generator_combos(copy_dict).items():
67                 for key2, value2 in generator_combos(comp_copy_dict).items():
68                     new_dict = {key1: value1, key2: value2 }
69                     result_dict = generator_combos(new_dict)
70                     output_dict.update(result_dict)
71                     count += len(result_dict)
72
73
74 return output_dict
```

Recursion

Part I: NYT Digits Solver (Easy Part)

Example:

```
In [9]: generator_combos({'1':1, '2': 2, '3': 3, '4': 4, '5':5 })
```

```
Out[9]: {'(((5+4)+3)+2)+1': 15,  
         '(((5+4)+3)+2)*1': 14,  
         '(((5+4)+3)+2)/1': 14,  
         '(((5+4)+3)+2)-1': 13,  
         '(((5+4)+3)*2)+1': 25,  
         '(((5+4)+3)*2)*1': 24,  
         '(((5+4)+3)*2)/1': 24,  
         '(((5+4)+3)*2)-1': 23,  
         '(((5+4)+3)/2)+1': 7,  
         '(((5+4)+3)/2)*1': 6,  
         '(((5+4)+3)/2)/1': 6,  
         '(((5+4)+3)/2)-1': 5,  
         '(((5+4)+3)-2)+1': 11,  
         '(((5+4)+3)-2)*1': 10,  
         '(((5+4)+3)-2)/1': 10,
```

Original Refined Question: How can we find all valid computational combinations of **any list of numbers** that yield a target number?

“If we already have the combinations, we can **easily** find the ones that match a given number. The **difficult** part is generating these combinations”

```
1 def nyt_digits_solver(num_list: list, target: int, soln_num=None):  
2  
3     # generating full list of combinations of all sizes for the input numbers  
4     soln_set_dict = dict()  
5     for i in range(2, len(num_list) + 1):  
6         for combo in list(combinations(num_list, i)):  
7             dict_conv = {str(num):num for num in list(combo)}  
8             soln_dict = generator_combos(dict_conv) ← “past 10 slides”  
9             soln_set_dict.update(soln_dict)  
10  
11     # enumerating all solutions (i.e., combinations that result in target)  
12     solns = [key for key in soln_set_dict if soln_set_dict[key]==target] ← “1 min”  
13  
14     # find the `soln_num`-shortest solutions (by char count)  
15     if soln_num:  
16         solns_copy = list()  
17         for k in sorted(solns, key=len)[:soln_num]:  
18             solns_copy.append(k)  
19         solns = solns_copy  
20  
21     return solns
```

[DEMO](#)

UPDATE Read the migration plan to Notebook 7 to learn about the new features and the actions to take if you are using extensions - Please note that updating to Notebook 7 might break some of your extensions. Don't show anymore

NYT Digits Puzzle - Algorithm

(May 23, 2023)

Problem Statement: Can you come up with some code that finds the solution to the NYT digits puzzle given six numbers and a target number?

```
In [1]: from itertools import combinations
from math import comb
```

```
In [2]: # generates all manipulation combinations of a dictionary of ele
def generator_combos(start_dict: dict):

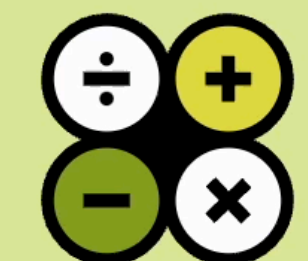
    # defining dict_len to determine pathing
    dict_len = len(start_dict)

    # defining output dictionary
    output_dict = dict()

    count = 0

    # output the dictionary if there is just one element
    if dict_len==1:
        output_dict = start_dict

    # output all ways to add, subtract, multiply, or divide two
```



Digits

BETA

A daily numbers puzzle from The New York Times.

This game is going away on August 8th.

Play

July 25, 2023

Part I: Hints at Something Deeper Solutions

When we count the number of generated combinations for various lists of numbers we find...

```
In [10]: for ell in range(1,7):  
         dict_ = {str(num):num for num in range(1,ell+1)}  
         all_combos = generator_combos(dict_)  
         print(f'Number of combinations with {ell} numbers:', len(all_combos))
```

```
Number of combinations with 1 numbers: 1  
Number of combinations with 2 numbers: 4  
Number of combinations with 3 numbers: 48  
Number of combinations with 4 numbers: 960  
Number of combinations with 5 numbers: 26880  
Number of combinations with 6 numbers: 967680
```

Is there a way that we can show why this is the case? And also use this to determine the complexity of this algorithm?

Part II: Mathematics

Part II: Counting Combinations

Part II

Mathematics

How can we count the number of solutions* to the puzzle?

$$F(x) = \sum_{k=0}^N x^k F_k$$

*We are actually counting the number of “generated combinations” and not exact solutions

*“closed-form” means we can compute values directly as a function

Refined Question:
How can we explain the following sequence?

```
In [10]: for ell in range(1,7):  
         dict_ = {str(num):num for num in range(1,ell+1)}  
         all_combos = generator_combos(dict_)  
         print(f'Number of combinations with {ell} numbers:', len(all_combos))
```

```
Number of combinations with 1 numbers: 1  
Number of combinations with 2 numbers: 4  
Number of combinations with 3 numbers: 48  
Number of combinations with 4 numbers: 960  
Number of combinations with 5 numbers: 26880  
Number of combinations with 6 numbers: 967680
```

Let's formulate this more abstractly

20. NYT Digits Puzzle

[combinatorics - numbers]

We have N integers $d_1 < d_2 < \dots < d_N$. We have four possible operations \times , \div , $+$, and $-$. Say that Q_N is the number of different real numbers (that are greater than 1) we can form from combining all N integers using the available operations.

(Definition) Q_N is the number of combinations created by this algorithm when there are N numbers (i.e., $Q_1 = 1$, $Q_2 = 4$, $Q_3 = 48$, etc.)

Further Refined Question: What is the closed-form* expression for Q_N ?

Part II: Counting Combinations

20. NYT Digits Puzzle

[combinatorics - numbers]

We have N integers $d_1 < d_2 < \dots < d_N$. We have four possible operations \times , \div , $+$, and $-$. Say that Q_N is the number of different real numbers (that are greater than 1) we can form from combining all N integers using the available operations.

Further Refined Question: What is the closed-form expression for Q_N ?

We will proceed as before (i.e., incrementally)

Two-Number Combinations:

$$d_2 + d_1, \quad d_2 - d_1, \quad d_2 \times d_1, \quad d_2/d_1$$

$$Q_2 = 4$$

Or with $Q_1 \equiv 1$

$$Q_4 = \frac{1}{2} Q_2 \left[\binom{4}{3} Q_3 Q_1 + \binom{4}{2} Q_2 Q_2 + \binom{4}{1} Q_1 Q_3 \right]$$

Three-Number Combinations:

$$Q_3 = Q_2 \binom{3}{2} Q_2 = 48$$

number of ways to combine the results of sub-combinations

number of ways to choose two numbers from three

number of ways to combine those two numbers

Four-Number Combinations:

$$Q_4 = Q_2 \binom{4}{3} Q_3 + \frac{1}{2} Q_2 \binom{4}{2} Q_2 Q_2 = 960$$

correction for double counting

number of ways to combine the results of sub-combinations

number of ways to choose two numbers from four

number of ways to combine those two chosen numbers

number of ways to combine the two unchosen numbers

Part II: Counting Combinations

20. NYT Digits Puzzle

[combinatorics - numbers]

We have N integers $d_1 < d_2 < \dots < d_N$. We have four possible operations \times , \div , $+$, and $-$. Say that Q_N is the number of different real numbers (that are greater than 1) we can form from combining all N integers using the available operations.

Further Refined Question: What is the closed-form expression for Q_N ?

We can use **dynamic programming** to generate solutions to this equation

N-Number Combinations:

$$Q_N = \frac{1}{2} Q_2 \sum_{k=1}^{N-1} \binom{N}{k} Q_{N-k} Q_k$$

Four-Number Combinations:

$$Q_1 \equiv 1$$

$$Q_4 = \frac{1}{2} Q_2 \left[\binom{4}{3} Q_3 Q_1 + \binom{4}{2} Q_2 Q_2 + \binom{4}{1} Q_1 Q_3 \right]$$

```
In [10]: for ell in range(1,7):
         dict_ = {str(num):num for num in range(1,ell+1)}
         all_combos = generator_combos(dict_)
         print(f'Number of combinations with {ell} numbers:', len(all_combos))

Number of combinations with 1 numbers: 1
Number of combinations with 2 numbers: 4
Number of combinations with 3 numbers: 48
Number of combinations with 4 numbers: 960
Number of combinations with 5 numbers: 26880
Number of combinations with 6 numbers: 967680
```

```
In [4]: for k in range(1, 7):
         print(f'QN for N={k}: {Q_calc(k)}')
```

QN for N=1: 1
QN for N=2: 4
QN for N=3: 48
QN for N=4: 960
QN for N=5: 26880
QN for N=6: 967680

```
1 from math import comb
2 import numpy as np
3
4 Q, Q2 = dict(), 4 # define dictionary and Q2 value
5 Q[1] = 1 # define first dictionary value
6
7 def Q_calc(N):
8
9     if N in Q.keys():
10         return Q[N]
11     else:
12         return (Q2//2)*np.sum([comb(N,k)*Q_calc(N-k)*Q_calc(k) for k in range(1,N)])
```

These solutions match the previous counting of results...

...however this expression is not in closed-form. We still need to solve for Q_N .

Part II: Closed form solution

20. NYT Digits Puzzle

[combinatorics - numbers]

We have N integers $d_1 < d_2 < \dots < d_N$. We have four possible operations $\times, \div, +,$ and $-$. Say that Q_N is the number of different real numbers (that are greater than 1) we can form from combining all N integers using the available operations.

Generating functions approach: Convert a recursive equation into an algebraic equation by summing over combinatorial coefficients

Further Refined Question: What is the closed-form expression for Q_N ?

$$Q_N = \frac{1}{2} Q_2 \sum_{k=1}^{N-1} \binom{N}{k} Q_{N-k} Q_k \quad \#1$$

...This expression is not in closed form. We still need to solve for Q_N .

This reproduces the series we found

A052714 $a(n) = 2^{(n-1)} * n! * \text{Catalan}(n-1)$ for $n > 0$ with $a(0) = 0$.
 0, 1, 4, 48, 960, 26880, 967680, 42577920, 2214051840, 132843110400, 9033331507200,
 686533194547200, 57668788341964800, 5305528527460761600, 530552852746076160000,
 57299708096576225280000, 6646766139202842132480000, 824199001261152424427520000 (list; graph; refs;
 listen; history; text; internal format)
 OFFSET 0, 3

<https://oeis.org/A052714>

"Generating Function" $Q(x) \equiv \sum_{\ell=1}^{\infty} \frac{x^\ell}{\ell!} Q_\ell$ and **#1**
#2

$$Q(x) = \frac{1}{Q_2} \left(1 - \sqrt{1 - 2Q_2 x} \right) \quad \leftarrow \text{Quadratic formula} \quad Q(x) - x = \frac{Q_2}{2} Q(x)^2$$

$$Q(x) = \sum_{k=1}^{\infty} C_{k-1} \left(\frac{Q_2}{2} \right)^{k-1} x^k$$

Taylor series expansion

$$C_n \equiv \frac{1}{n+1} \binom{2n}{n}$$

Catalan Numbers

Which (with **#2**) gives us

$$Q_N = N! \left(\frac{Q_2}{2} \right)^{N-1} C_{N-1}$$

See "[NYT Digits Puzzle - Mathematics](#)" for derivation details

Part III: Complexity

Part III: Complexity Class

Part III

Complexity Class

What is the computational complexity class of this problem?

$$P \subseteq NP \subseteq \text{EXP-TIME}$$

20. NYT Digits Puzzle

[combinatorics - numbers]

We have N integers $d_1 < d_2 < \dots < d_N$. We have four possible operations \times , \div , $+$, and $-$. Say that Q_N is the number of different real numbers (that are greater than 1) we can form from combining all N integers using the available operations.

$$Q_N = N! \left(\frac{Q_2}{2} \right)^{N-1} C_{N-1},$$

$$C_n \equiv \frac{1}{n+1} \binom{2n}{n}$$

Catalan Numbers

$$Q_N = \left(\frac{Q_2}{2} \right)^{N-1} \frac{(2N-2)!}{(N-1)!}$$

With Stirling's approximation
 $N! \sim (N/e)^N$

$$Q_N \sim O((2Q_2)^N)$$

$$Q_2 = 4$$

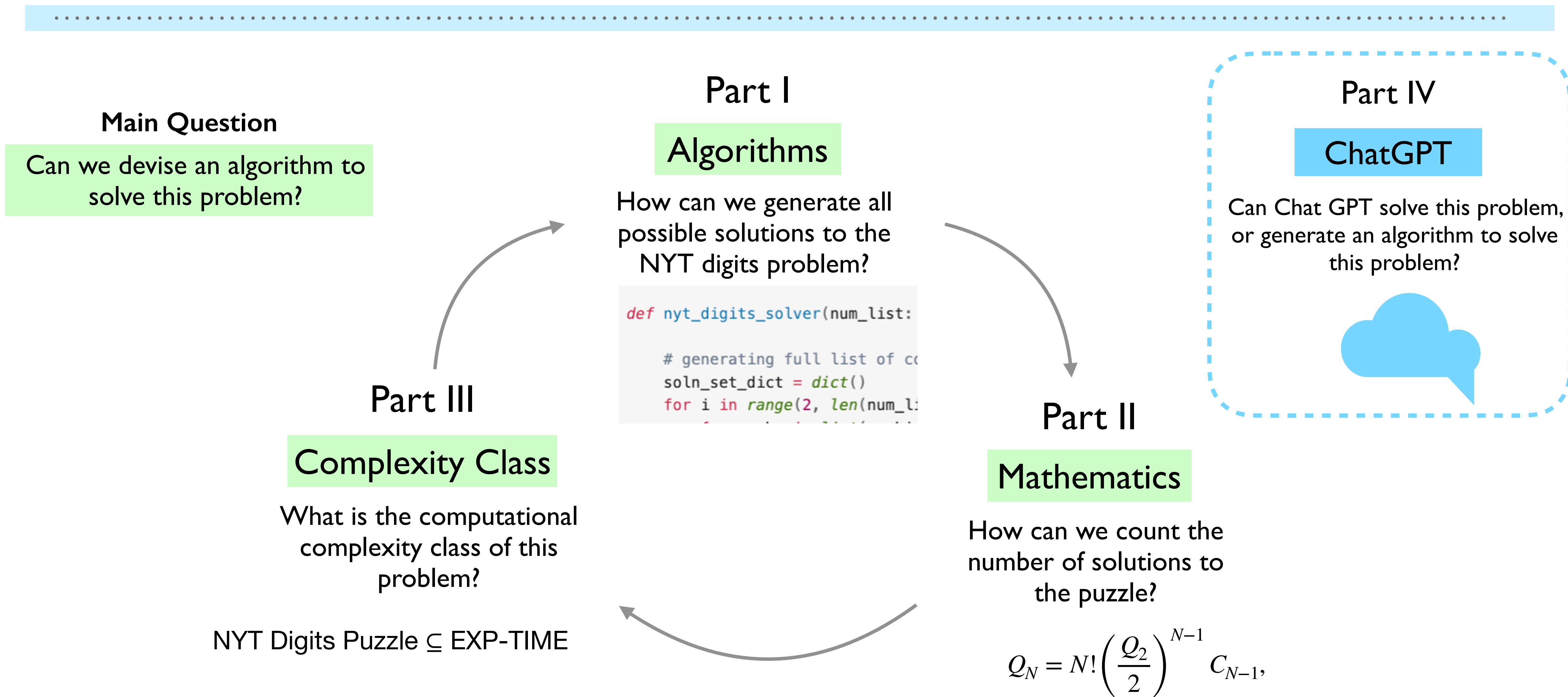
Exponential Time Computation

$$Q_N \sim O(2^{3N})$$

"As N increases it eventually becomes impossible to list every solution"

NYT Digits Puzzle \subseteq EXP-TIME

Presentation Outline



Part IV: ChatGPT

General Algorithm

Use any combination of numbers to reach the target:

52

1 2 3

4 5 25

↺ + - × ÷

Submit

1. You are given six numbers and a target number.
2. The goal is to reproduce the target number from any combination from the set of six numbers using the operations addition, subtraction, division, and multiplication.
3. In combining numbers, divisions cannot have remainders and subtractions cannot result in negative numbers.

Part IV

ChatGPT

Can Chat GPT solve this problem, or generate an algorithm to solve this problem?



[DEMO](#)

ChatGPT Home Page - Select algorithm_clean - J

chat.openai.com

New chat

GPT-3.5 GPT-4

- "Explain quantum computing in simple terms" →
- "Got any creative ideas for a 10 year old's birthday?" →
- "How do I make an HTTP request in Javascript?" →

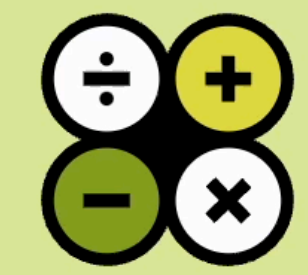
Capabilities

- Remembers what user said earlier in the conversation
- Allows user to provide follow-up corrections
- Trained to decline inappropriate requests

We are going to play a game. In this game there is a list of six numbers and also a target number. The objective of the game is to reproduce the target number from any combination from the set of six numbers using the operations addition, subtraction, division, and multiplication. In combining numbers, divisions cannot have remainders and subtractions cannot result in negative numbers. Do you understand the rules of the game?

Digits: A Daily Math Puzzle - The New York Times

nytimes.com/games...



Digits BETA

A daily numbers puzzle from The New York Times.

This game is going away on **August 8th**.

Play

July 25, 2023

GPT-3.5 GPT-4

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Capabilities

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In this game there is a list of six numbers and also a target number. The objective of the game is to reproduce the target number from any combination of the set of six numbers using the operations addition, subtraction, division, and multiplication. In combining numbers, divisions cannot have remainders and subtractions cannot result in negative numbers. Please devise an algorithm written in python that can generate the series of operations that must be applied to a list of numbers `num_list` in order to obtain the target integer `target`.

Moving Up 3

81 ☆☆☆	106 ☆☆☆	238 ☆☆☆	384 ☆☆☆	413 ☆☆☆
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Your progress ...

Use any combination of numbers to reach the target:

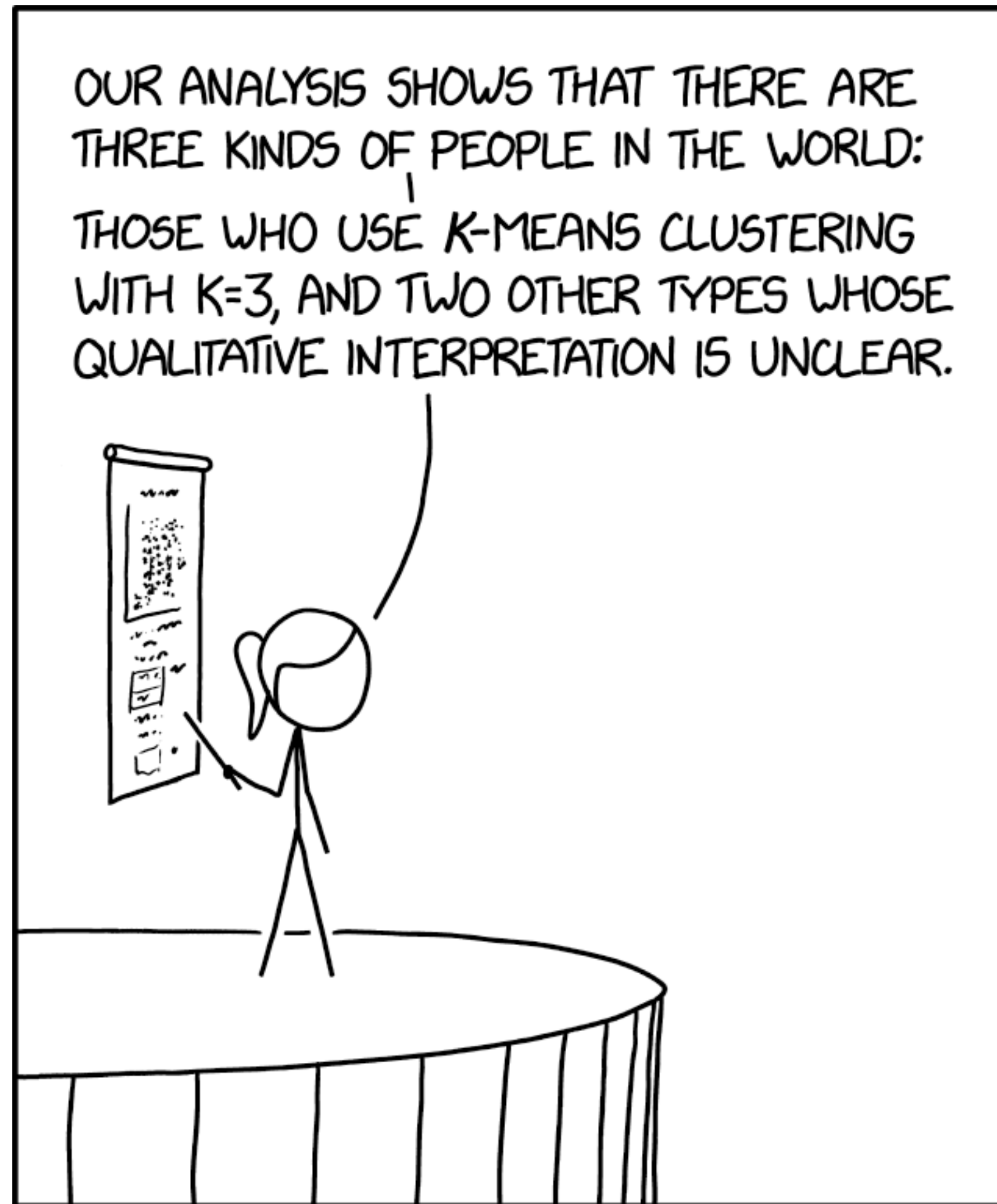
413

5	11	19
20	23	25

↺ + - × ÷

Submit

Requisite XKCD comic



<https://xkcd.com/2731/>

Thanks for listening!

References

<https://mobowill.com/nyt-digits-puzzle-solution-algorithm/>

<https://mobowill.com/nyt-digits-puzzle-mathamtics/>

https://math.mit.edu/~rstan/transparencies/miami_catalan.pdf

[\[JFR Research Day\] Programmatic Solution to NYT Connections](#)